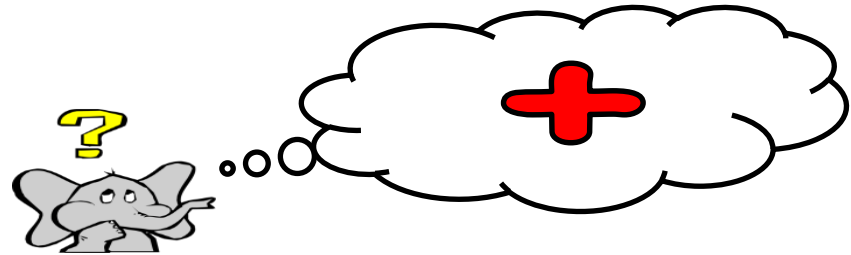


Addition

Progression in Teaching Addition

Mental Skills

Recognise the size and position of numbers
Count on in ones and tens
Know number bonds to 10 and 20
Add multiples of 10 to any number
Partition and recombine numbers
Bridge through 10

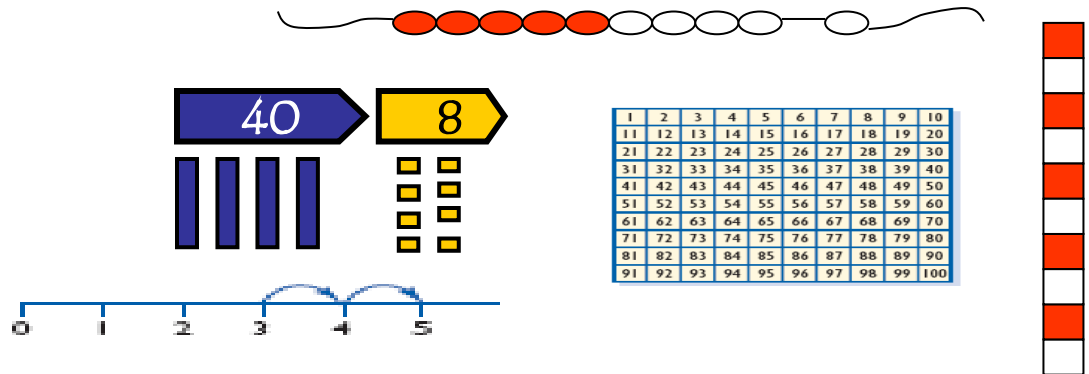


Models and Images

Counting apparatus
Place value apparatus
Place value cards
Number tracks
numicon
Numbered number lines
Marked but unnumbered number lines
Empty number lines
Hundred square
Counting stick
Bead string
Models and Images charts
ITPs - Number Facts, Ordering Numbers, Number Grid, Counting on and back in ones and tens

Key Vocabulary

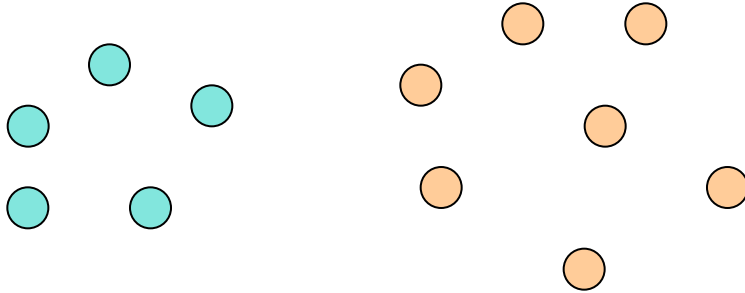
add
addition
plus
and
count on
greater
more
Most
sum
total
altogether
increase



add and count on
addition plus
more sum total
altogether increase

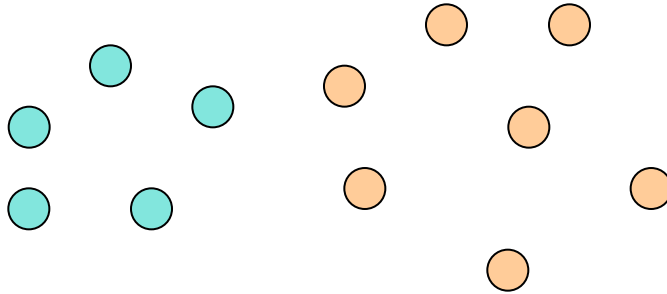
Models for addition

Combining two sets of objects (aggregation)



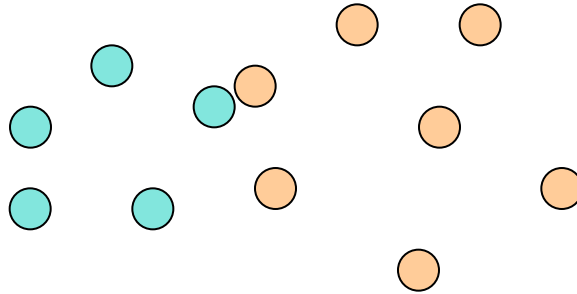
Models for addition

Combining two sets of objects (aggregation)



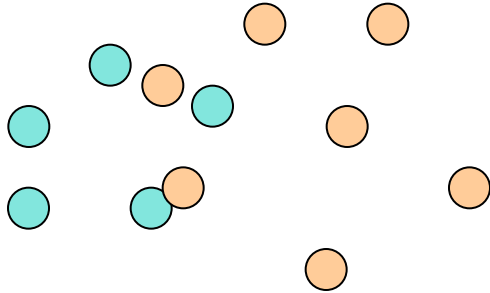
Models for addition

Combining two sets of objects (aggregation)



Models for addition

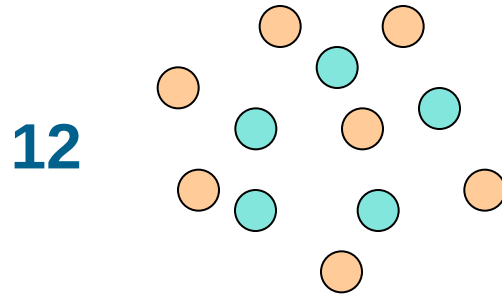
Combining two sets of objects (aggregation)



Models for addition

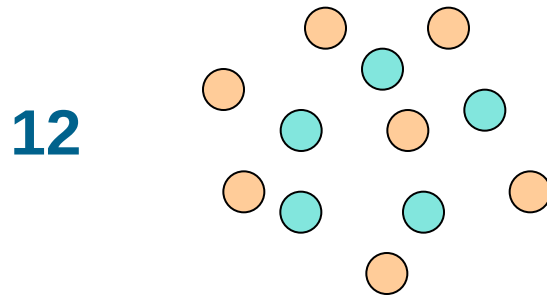


Combining two sets of objects (aggregation)



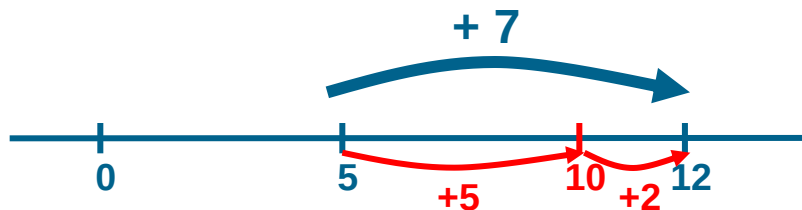
Tend to count one set, count the other and then count all.

Adding on to a set (augmentation)



Requires fluency with counting from any number.

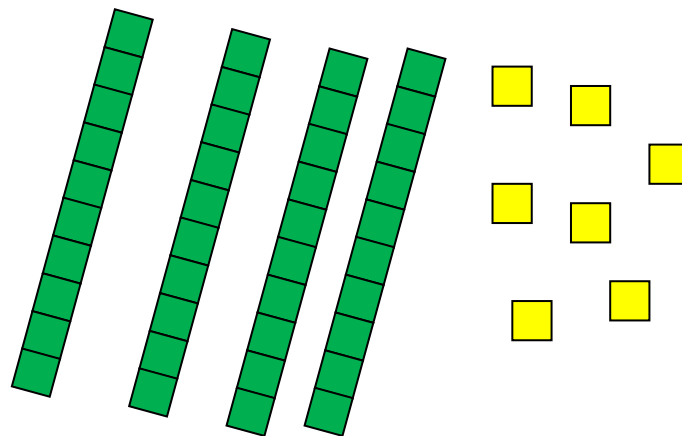
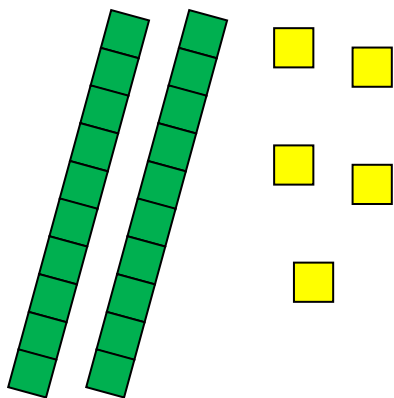
Counting on with a bead bar/number line

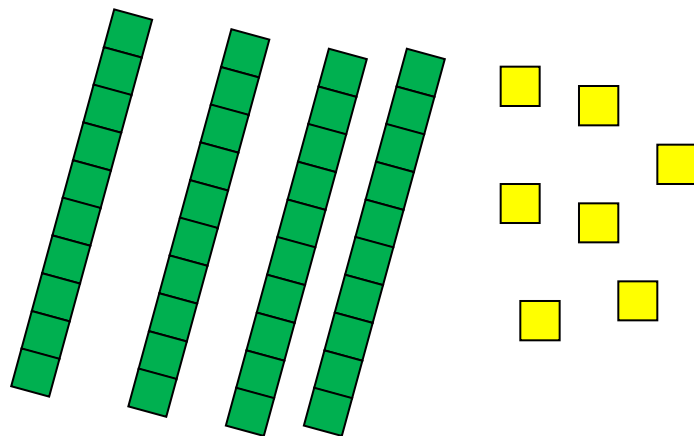
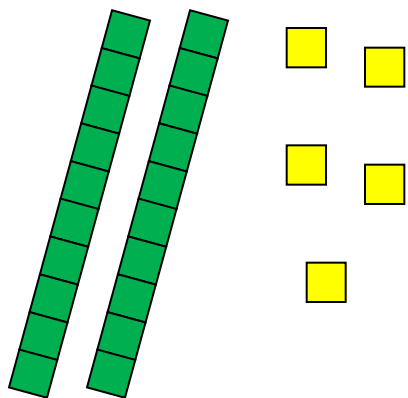


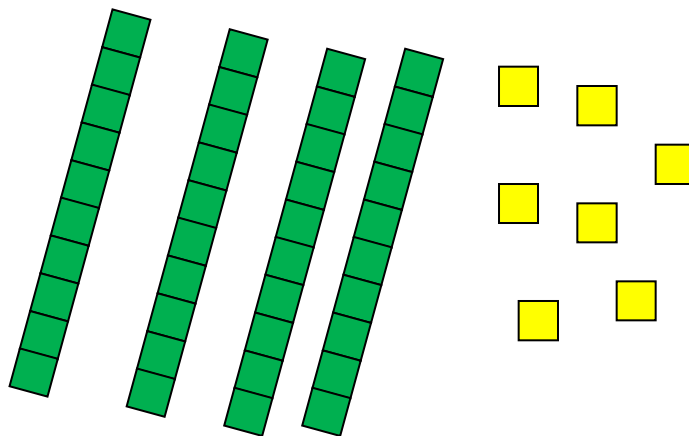
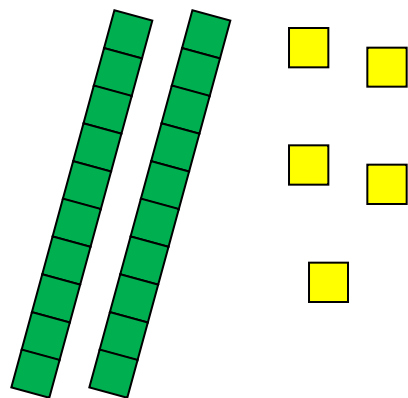
Also:

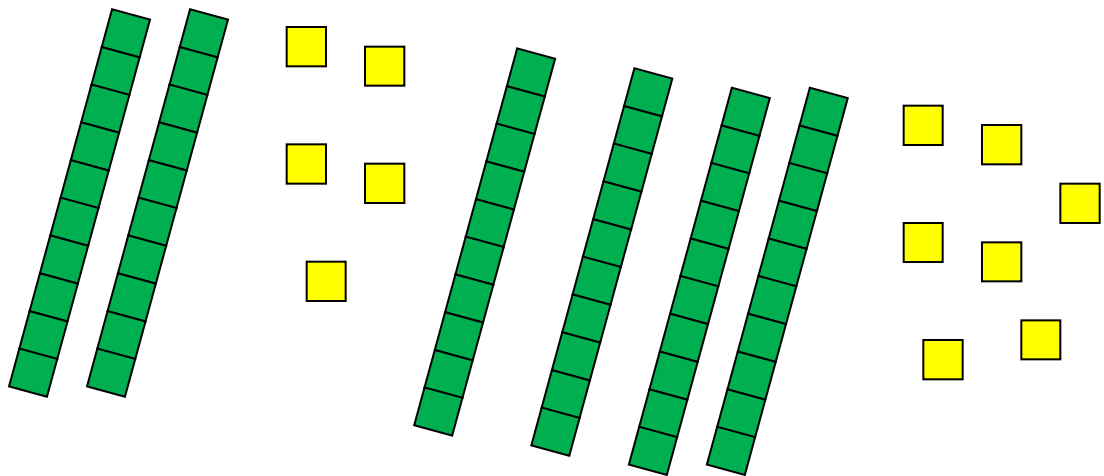
Bead bar and number line (showing 10s) encourages use of number bonds and place value for added efficiency⁷

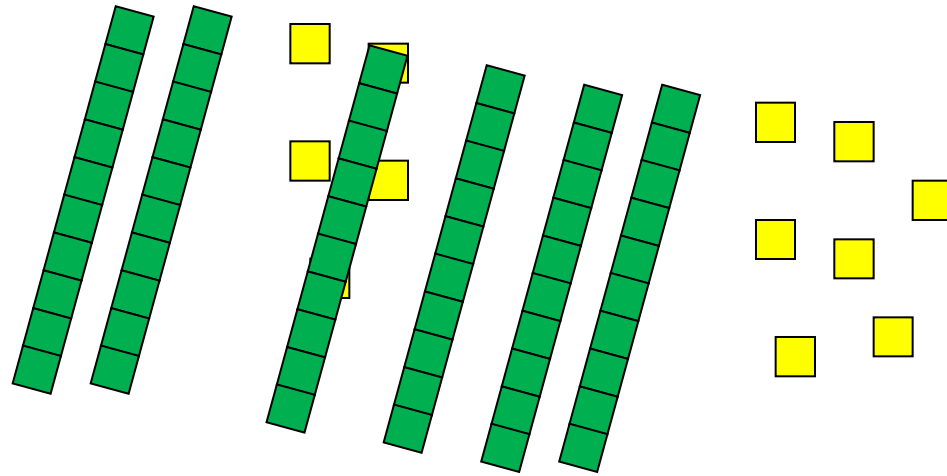
More than single digits?

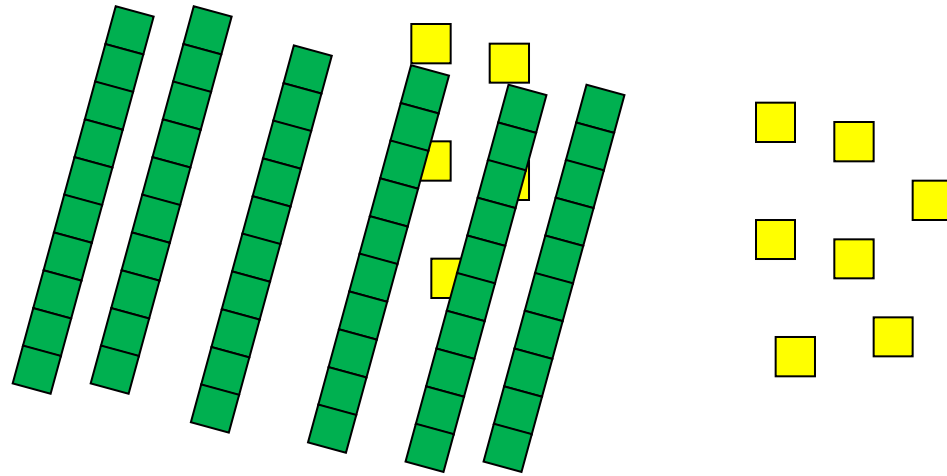


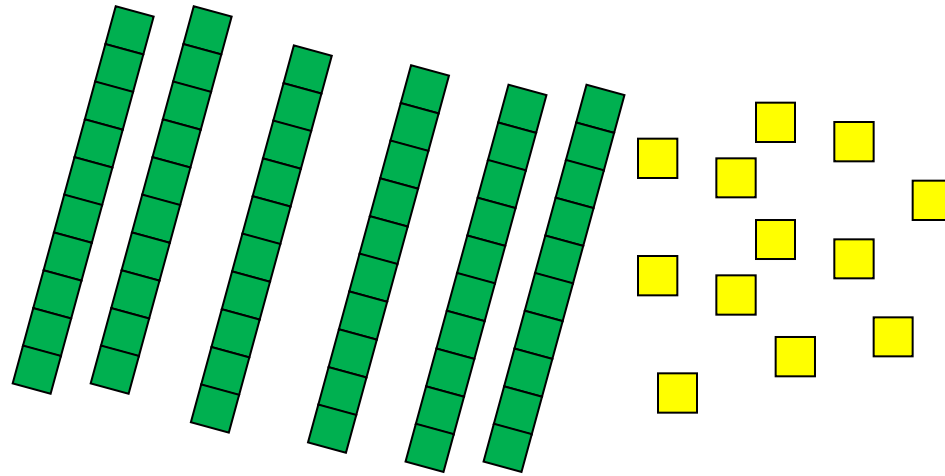


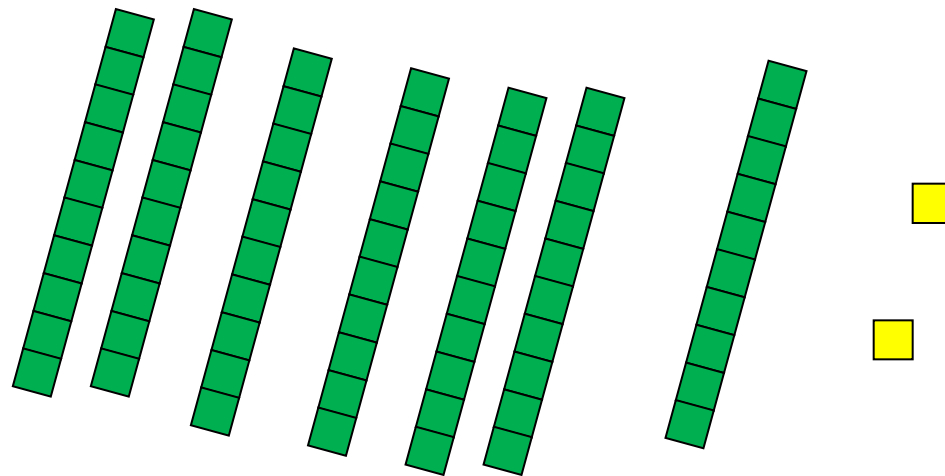


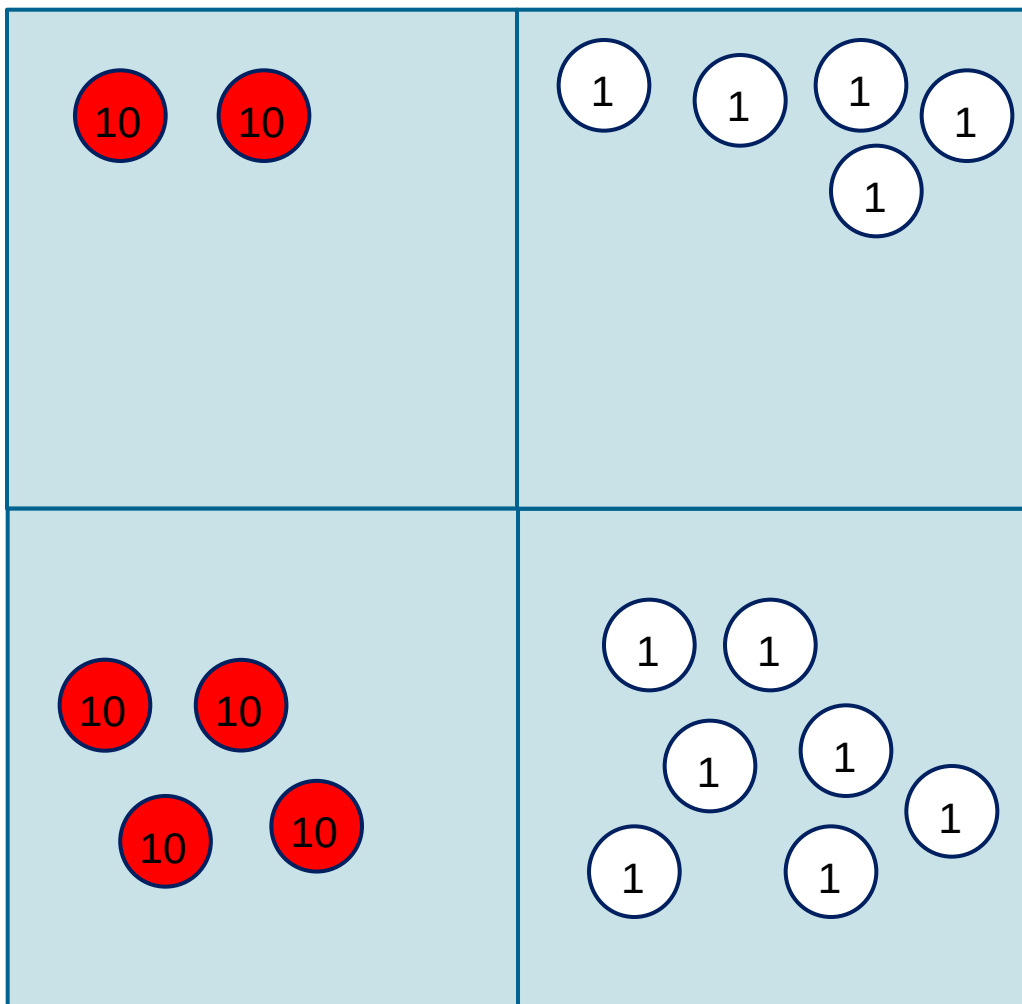




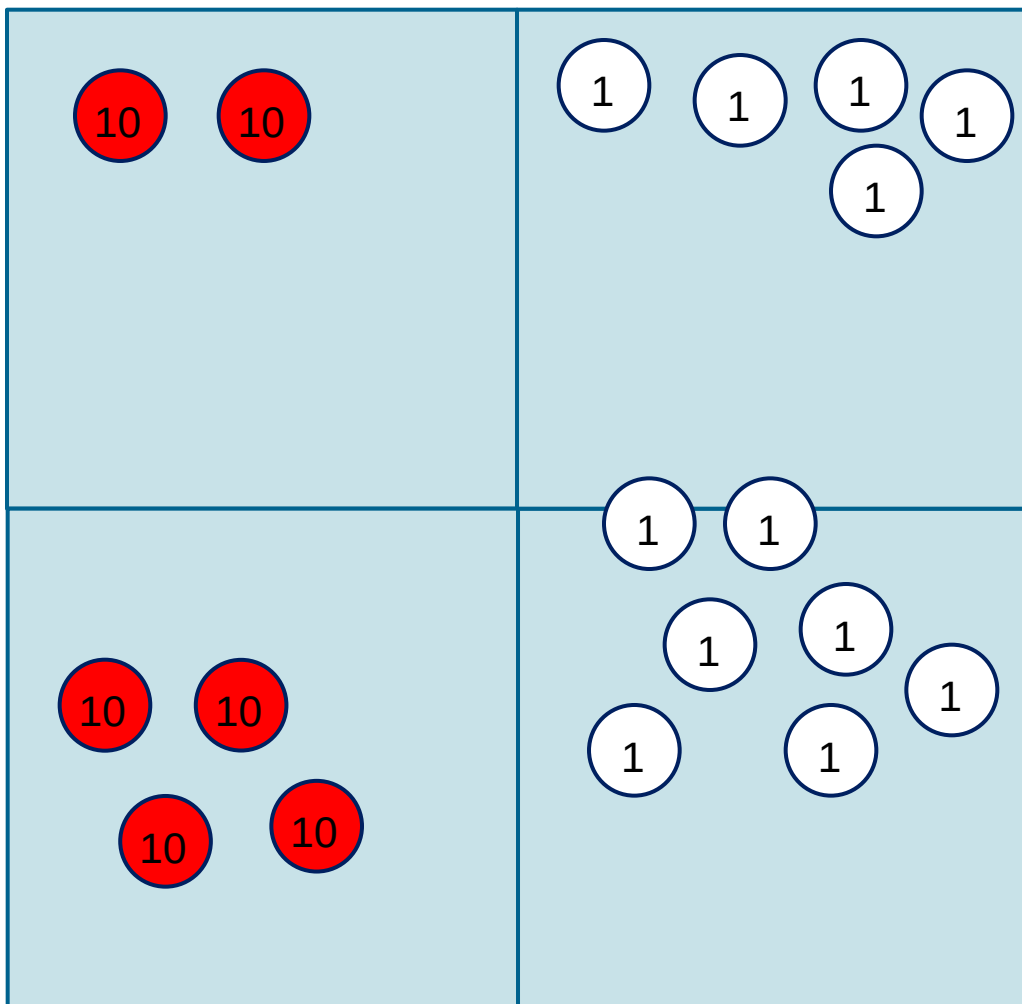




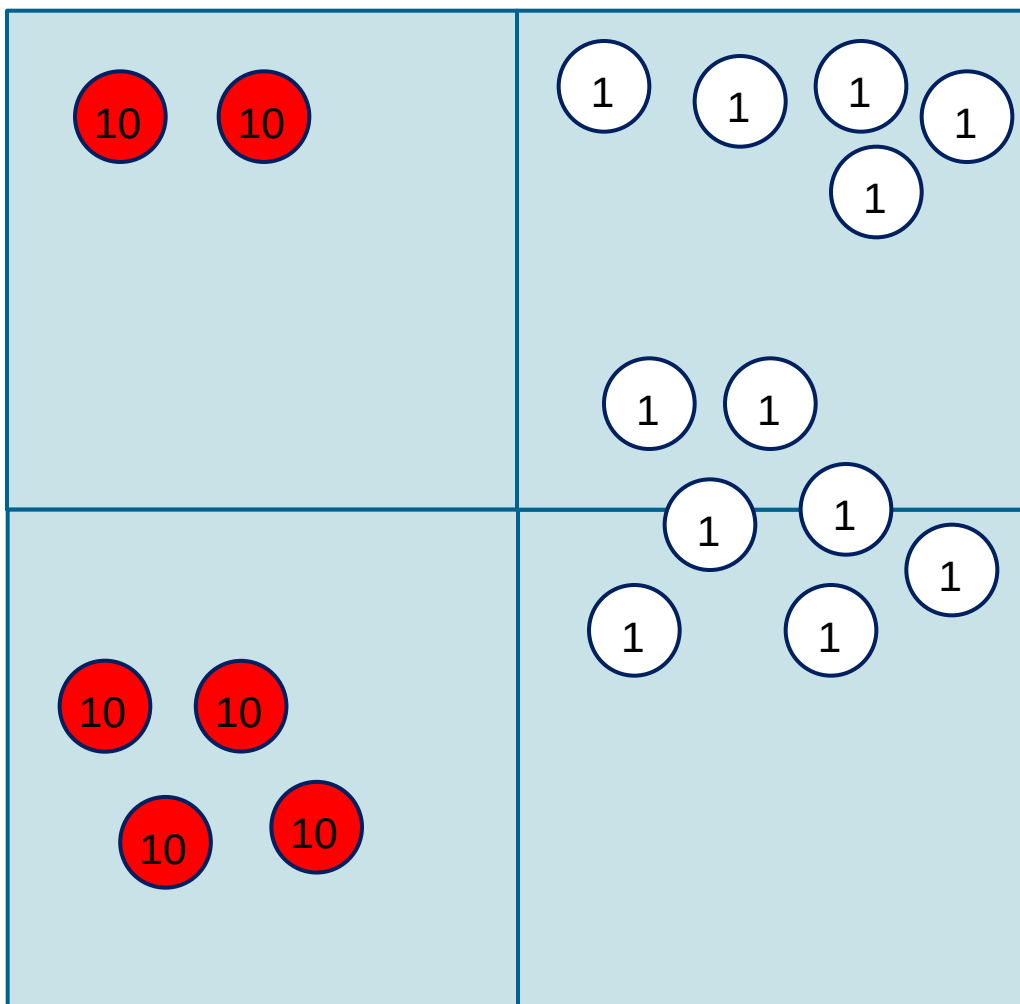




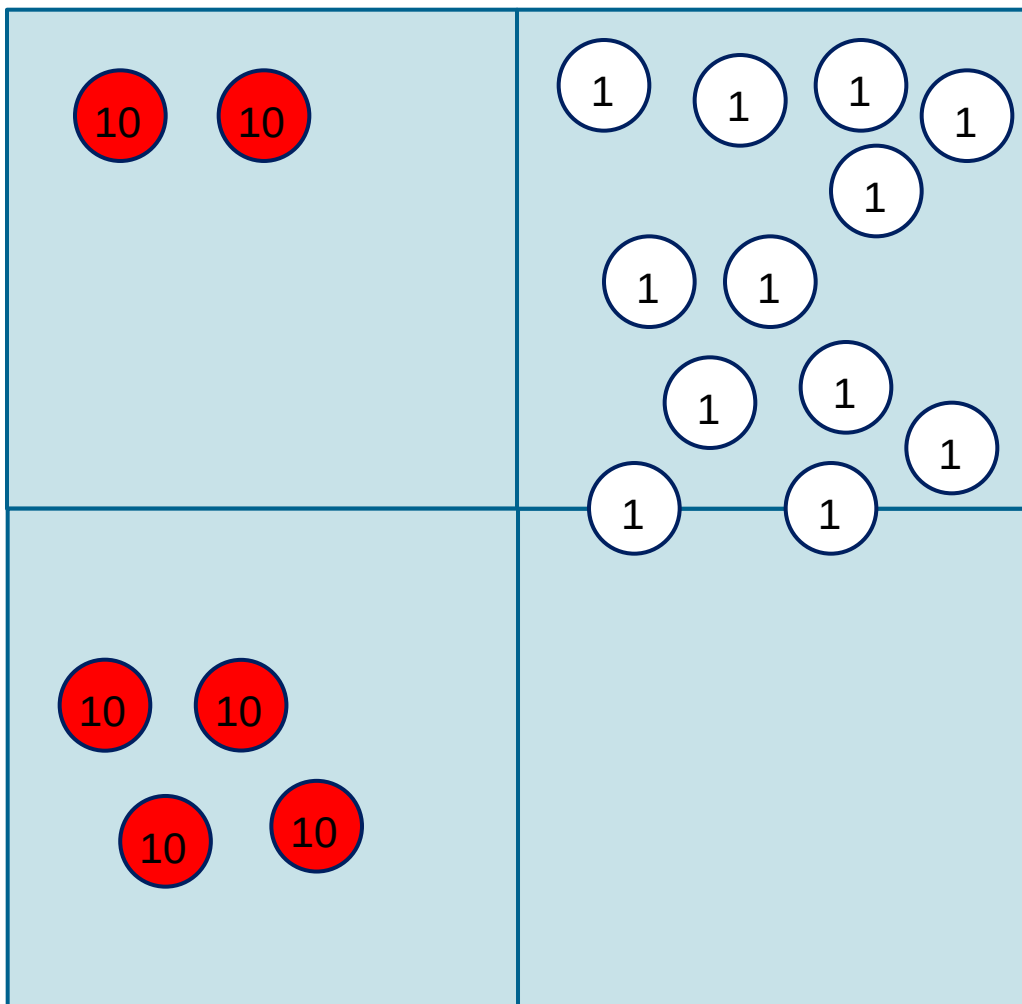
$$\begin{array}{r} 25 \\ +47 \\ \hline \\ \hline \end{array}$$



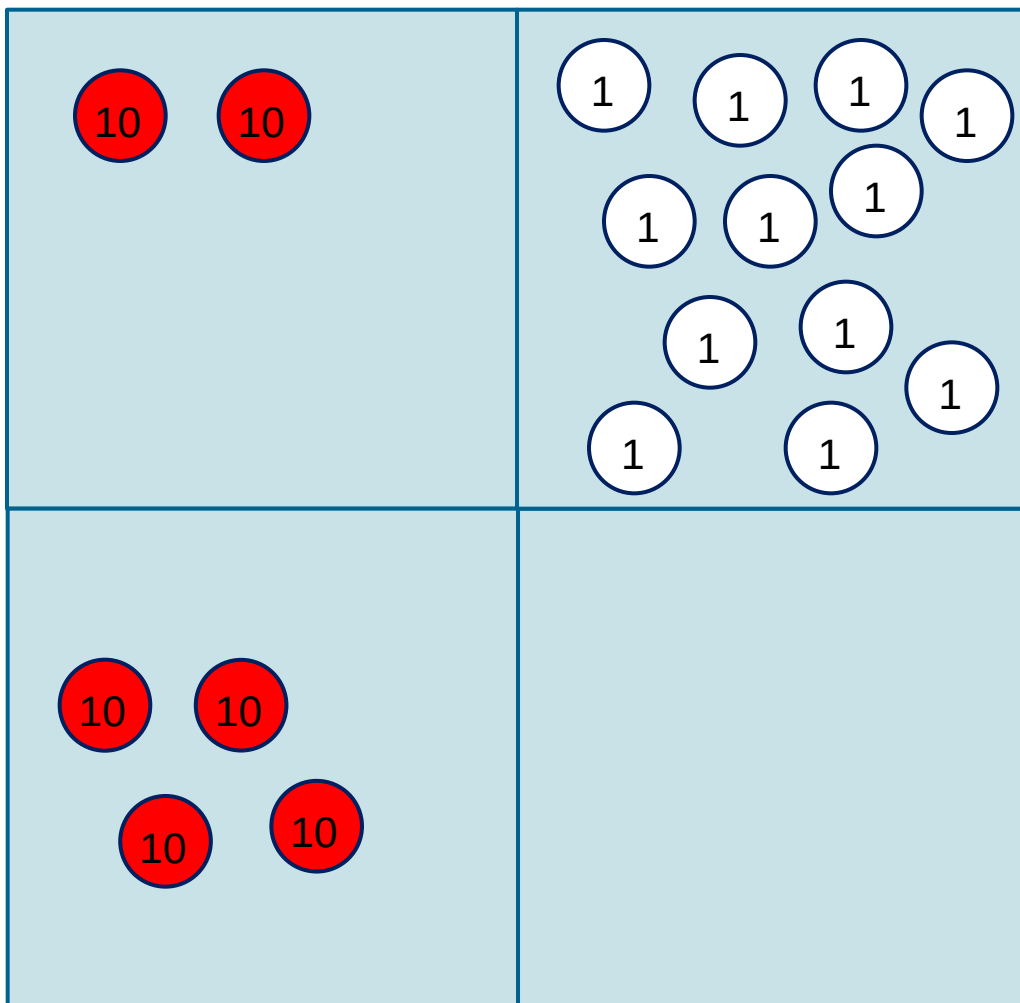
$$\begin{array}{r} 25 \\ +47 \\ \hline \\ \hline \end{array}$$



$$\begin{array}{r} 25 \\ +47 \\ \hline \\ \hline \end{array}$$

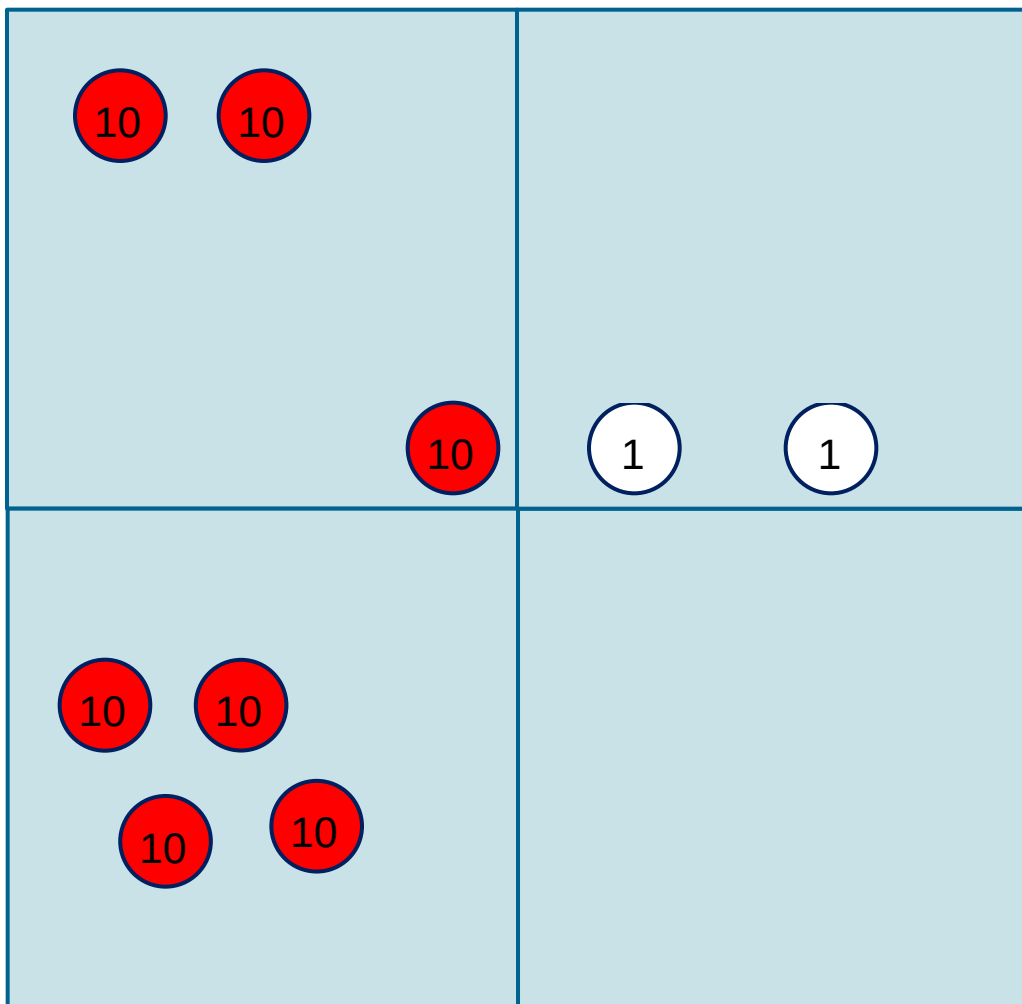


$$\begin{array}{r} 25 \\ +47 \\ \hline \\ \hline \end{array}$$

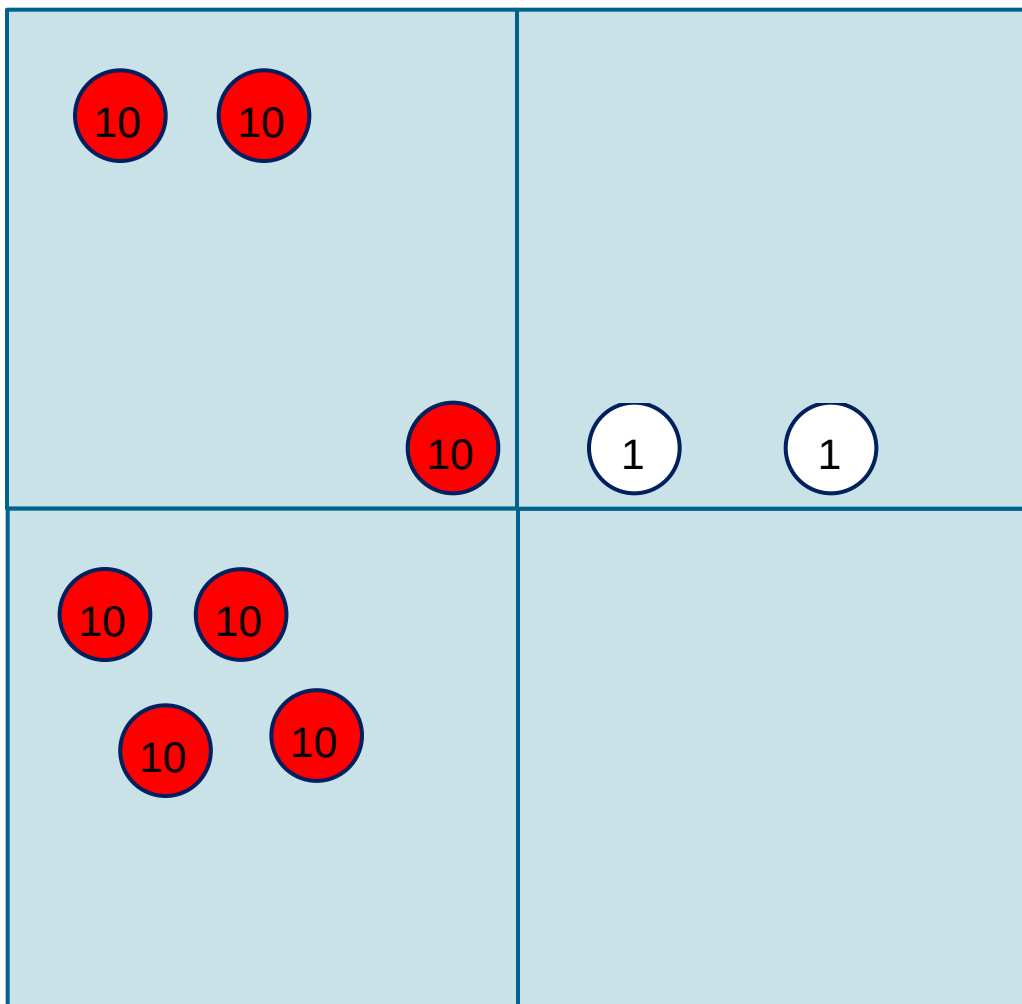


$$\begin{array}{r} 25 \\ +47 \\ \hline \end{array}$$

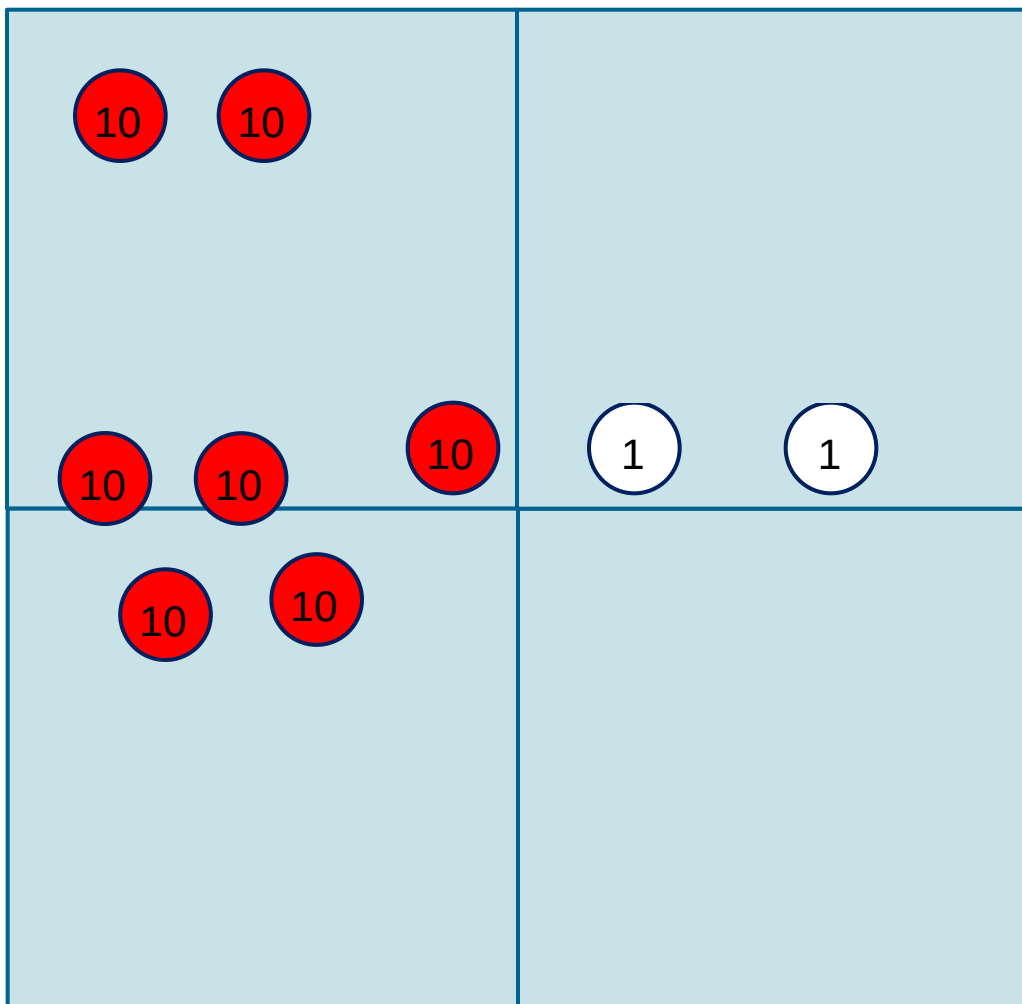
12



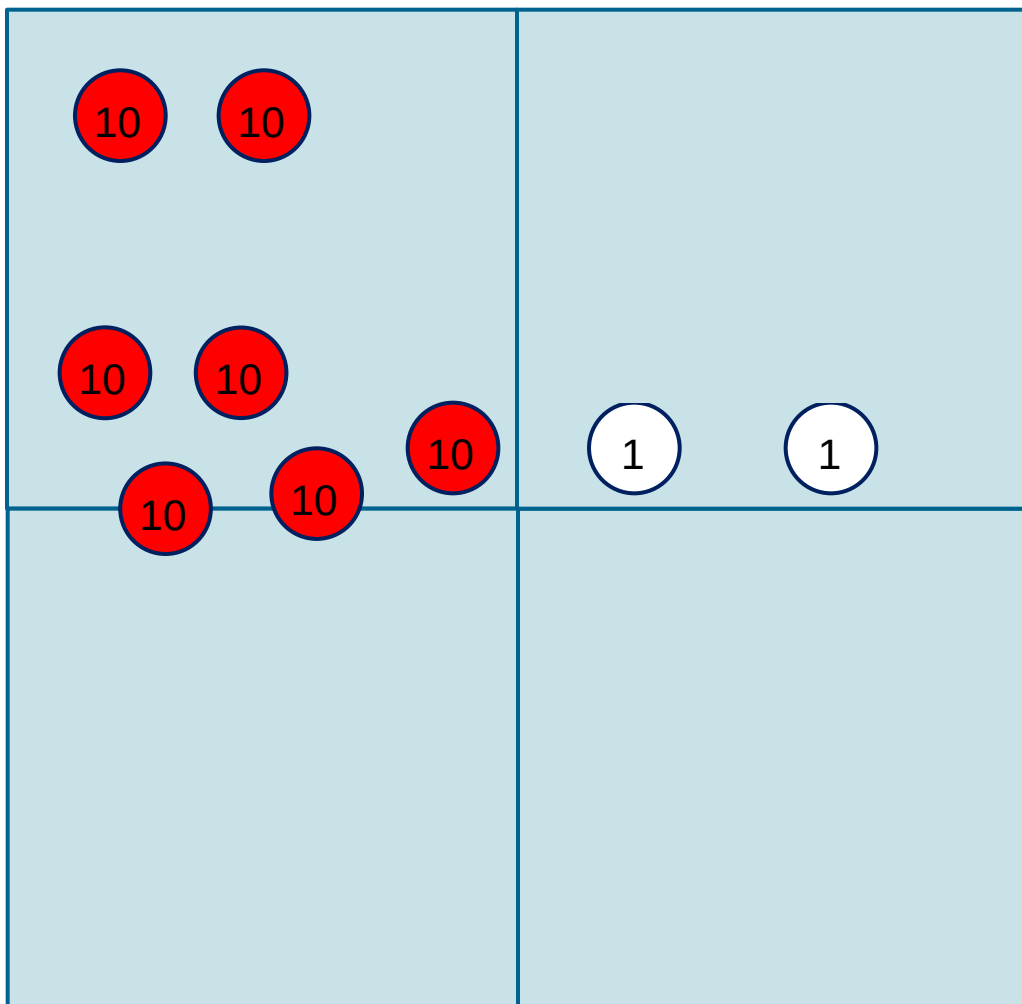
$$\begin{array}{r} 25 \\ +47 \\ \hline 2 \\ \hline 1 \end{array}$$



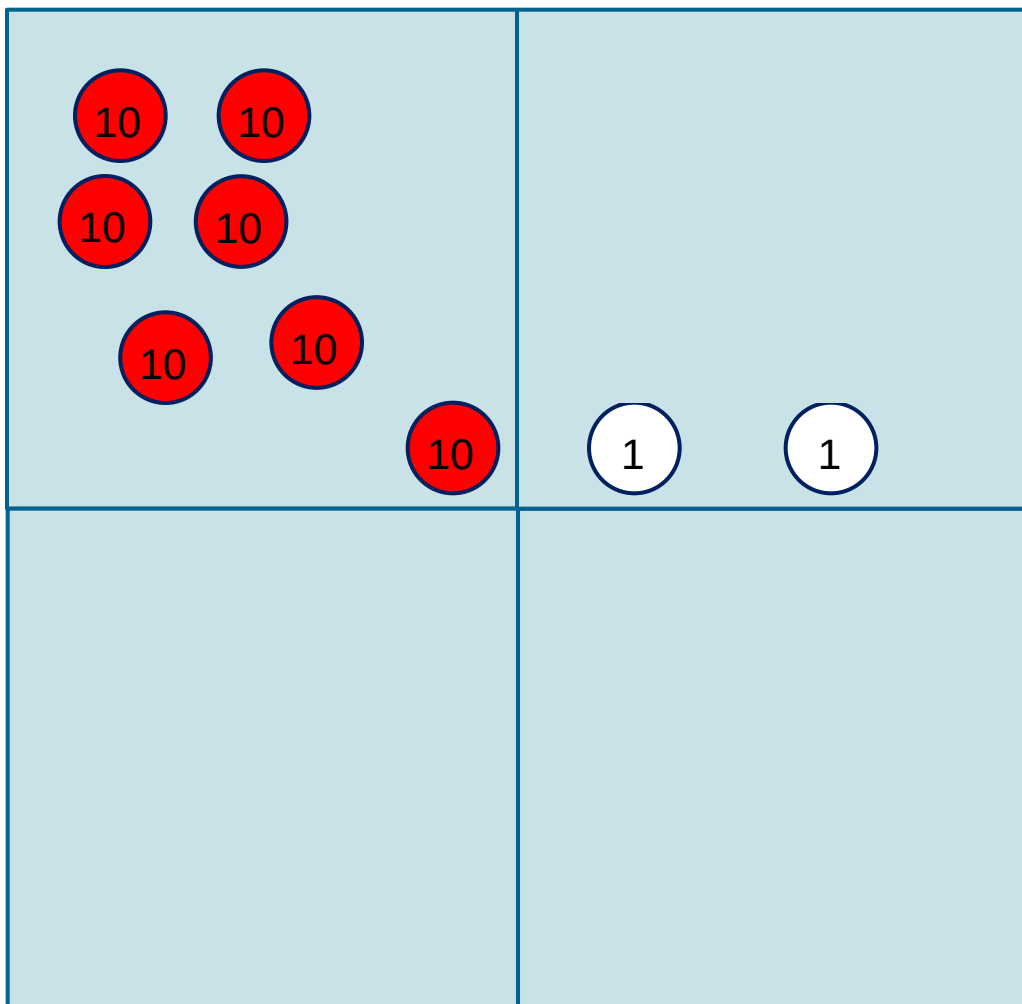
$$\begin{array}{r} 25 \\ +47 \\ \hline 2 \\ \hline 1 \end{array}$$



$$\begin{array}{r} 25 \\ +47 \\ \hline 2 \\ \hline 1 \end{array}$$

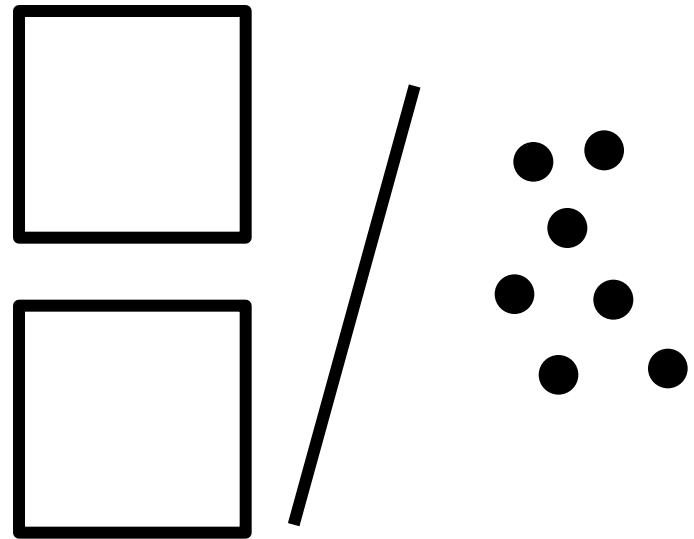
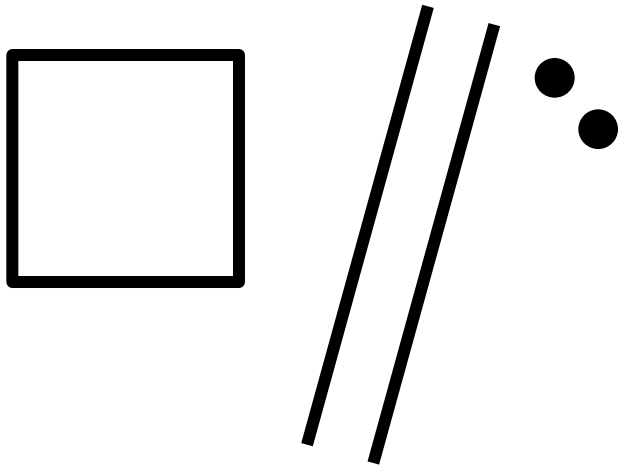


$$\begin{array}{r} 25 \\ +47 \\ \hline 2 \\ \hline 1 \end{array}$$

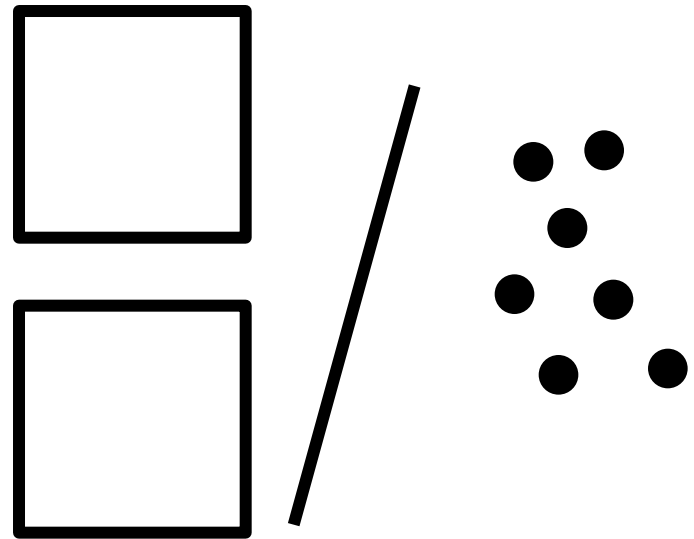
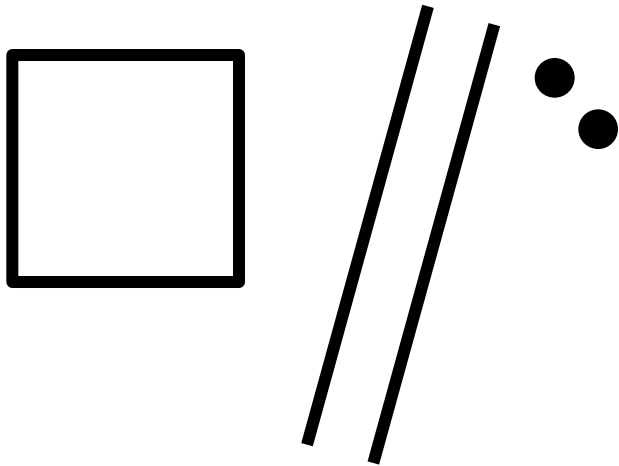


$$\begin{array}{r} 25 \\ +47 \\ \hline 72 \\ \hline 1 \end{array}$$

Children can support their own calculations by using jottings, e.g. $122 + 217$



Children can support their own calculations by using jottings, e.g. $122 + 217$

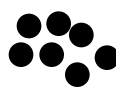


T U

6 7

+ 2 4

1 1

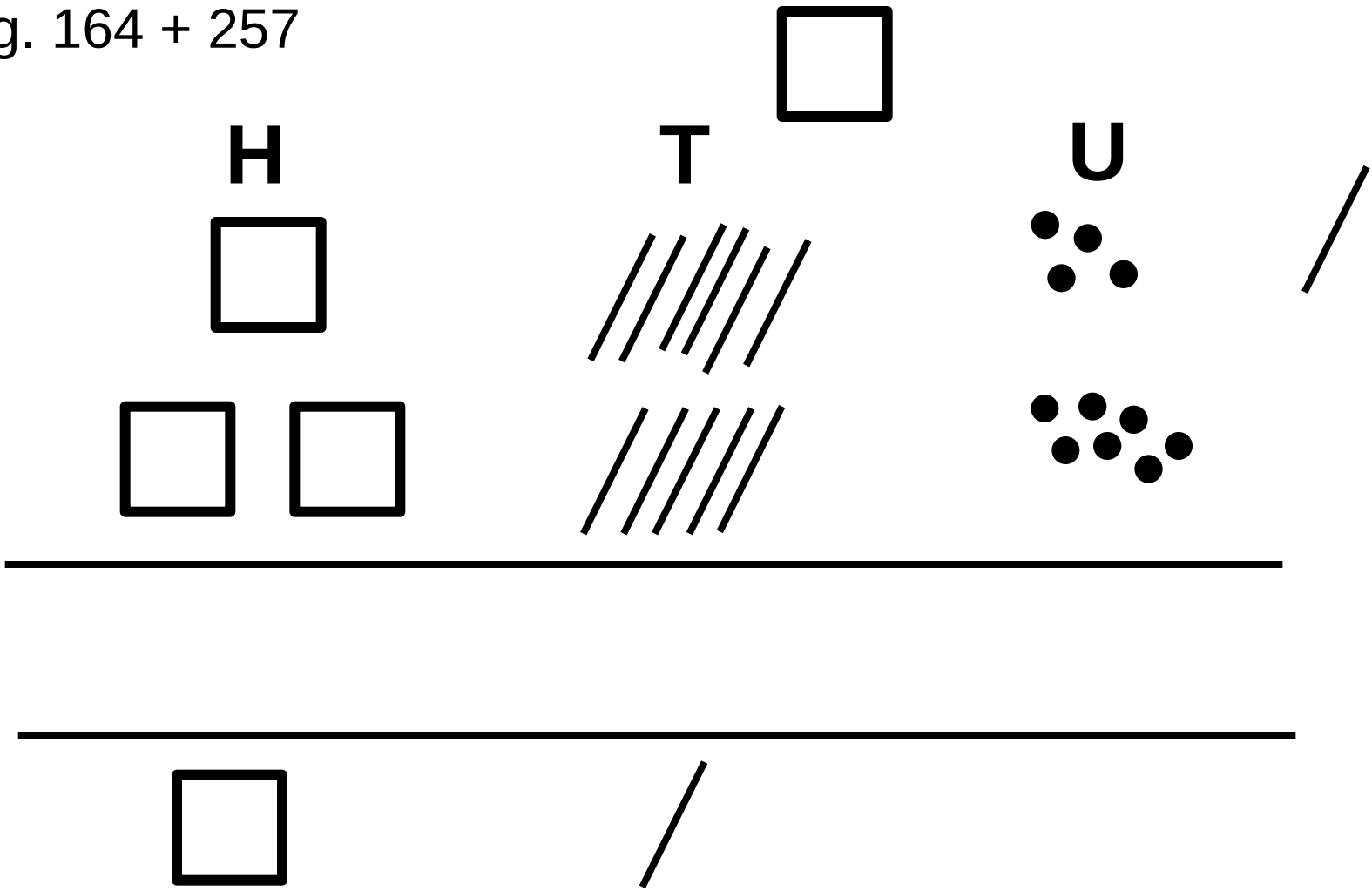


8 0



9 1

e.g. 164 + 257



H T U

1 6 4

+ 2 5 7

4 2 1

1 1

Subtraction

Progression in Teaching Subtraction

Mental Skills

Recognise the size and position of numbers

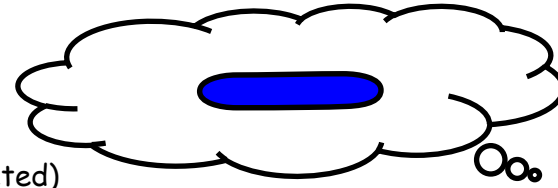
Count back in ones and tens

Know number facts for all numbers to 20

Subtract multiples of 10 from any number

Partition and recombine numbers (only partition the number to be subtracted)

Bridge through 10



Models and Images

Counting apparatus

Place value apparatus

Place value cards

Number tracks

Numbered number lines

Marked but unnumbered lines

Hundred square

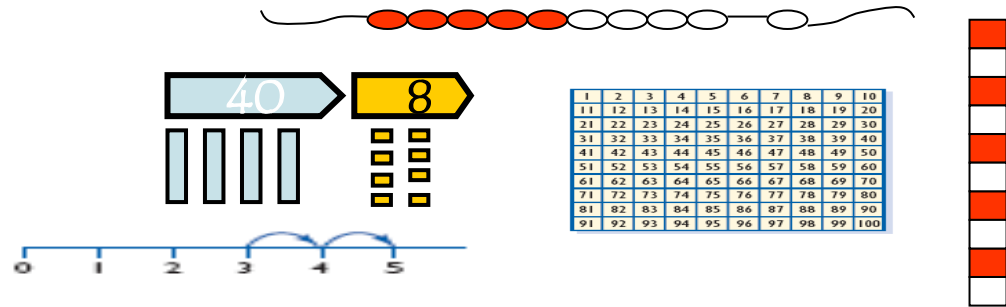
Empty number lines.

Counting stick

Bead strings

Models and Images Charts

ITPs - Number Facts, Counting on and back in ones and tens, Difference



Key Vocabulary

Subtract

take away (specific use)

minus

count back

less

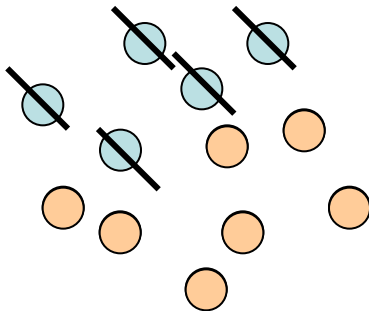
fewer

difference between

count back take away
fewer subtract
minus less
difference between

Models for subtraction

Removing items from a set (reduction or take-away)

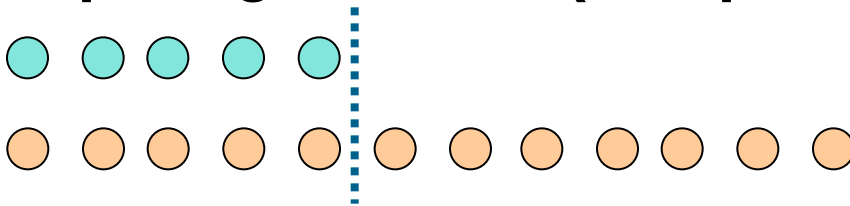


$$12 - 5 = 7$$

N.B.

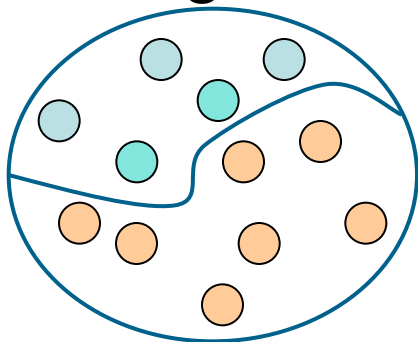
When this is done on a bead bar, there are links with both counting back and difference on a number line

Comparing two sets (comparison or difference)



Useful when two numbers are 'close together', where 'take-away' image can be cumbersome

Seeing one set as partitioned



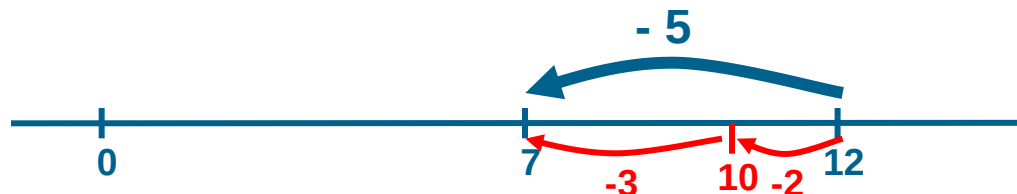
Seeing 12 as
made up of 5
and 7



Helps to see the related calculations;
 $5+7=12$, $7+5=12$, $12-7=5$ and $12-5=7$ as
all in the same diagram

Models for subtraction

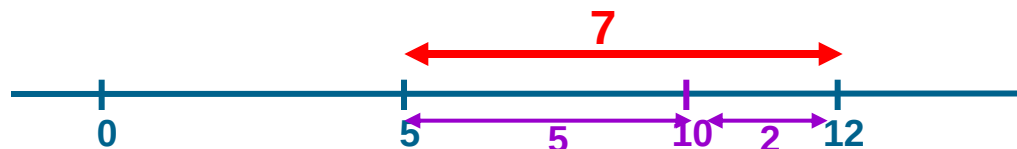
Counting back on a number line



Also:

Knowledge of place value and number bonds can support more efficient calculating

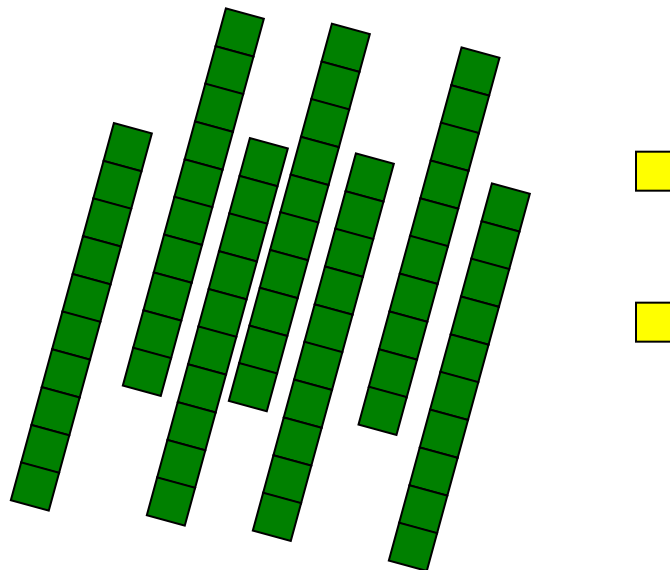
Finding the difference on a number line



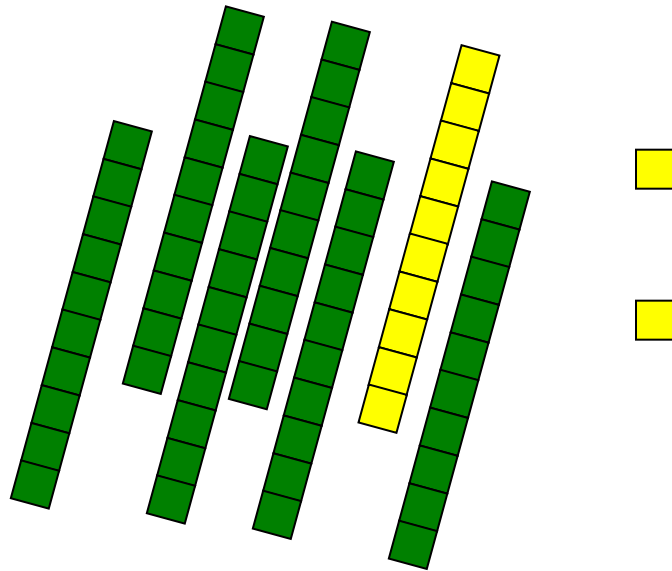
Useful when two numbers are 'close together', use of number bonds and place value can help.

More than single digits?

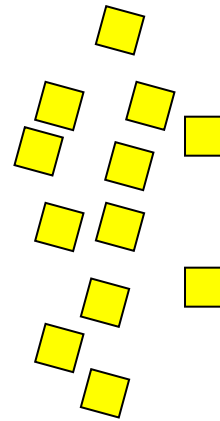
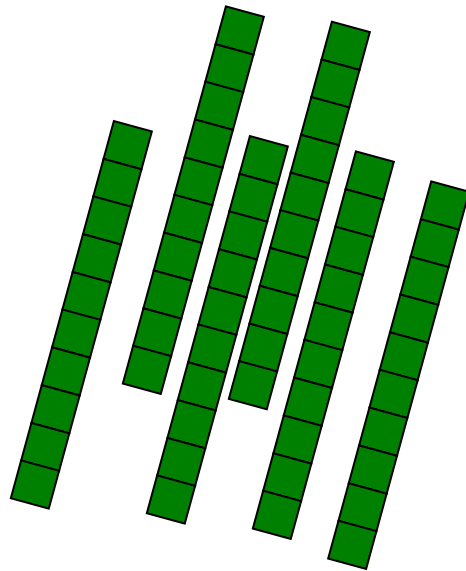
72 - 47



$$72 - 47$$



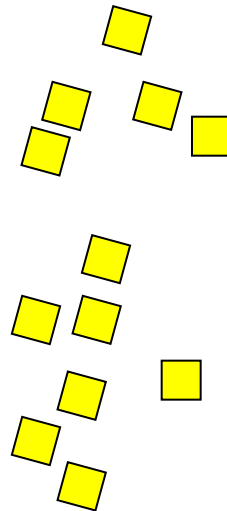
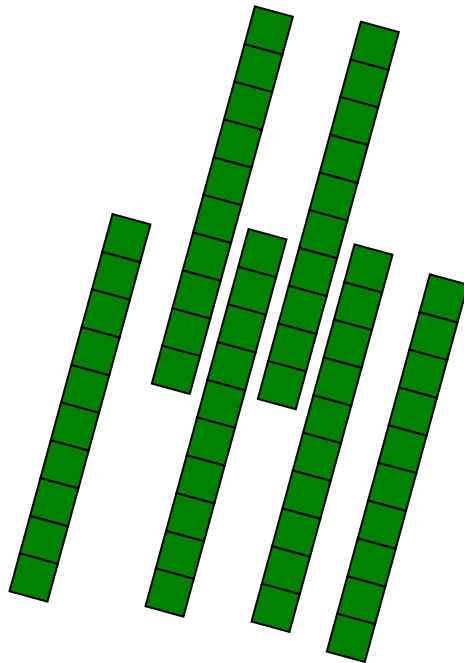
$$72 - 47$$



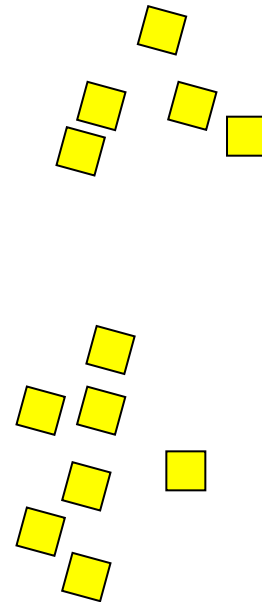
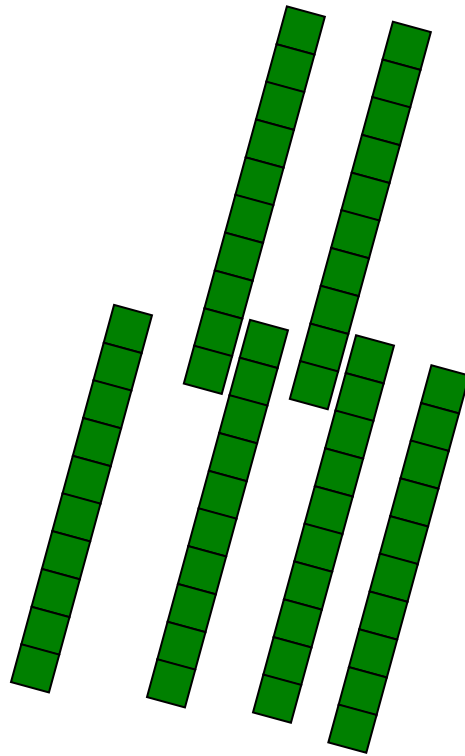
This is now “Sixty-twelve”

$$\begin{array}{r} 6 \\ \cancel{\nearrow} \end{array} \begin{array}{r} 1 \\ \cancel{\nearrow} \end{array} 2$$

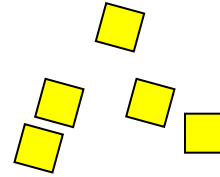
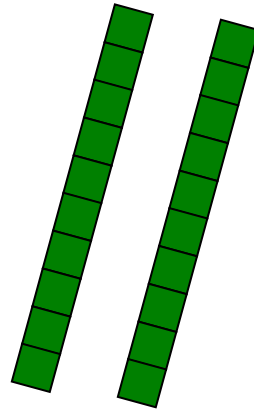
$$72 - 47$$

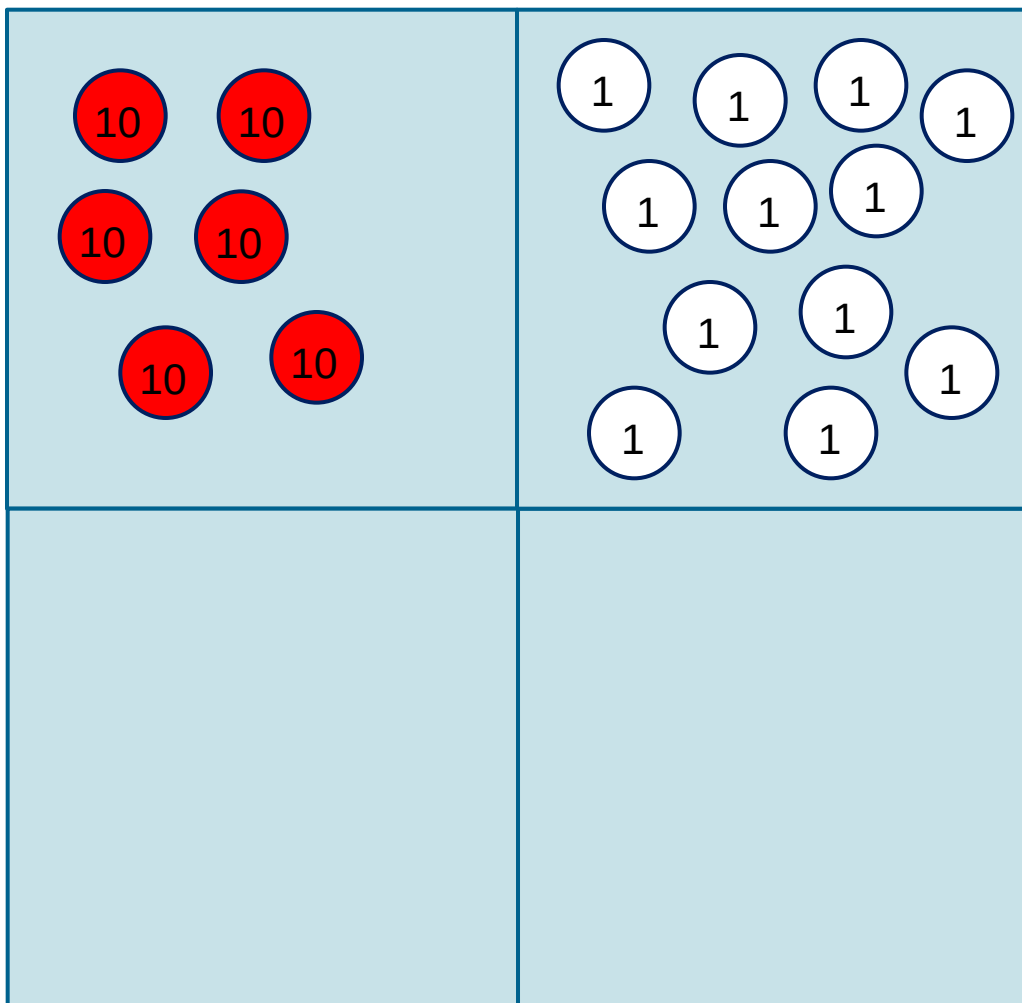


$$72 - 47$$

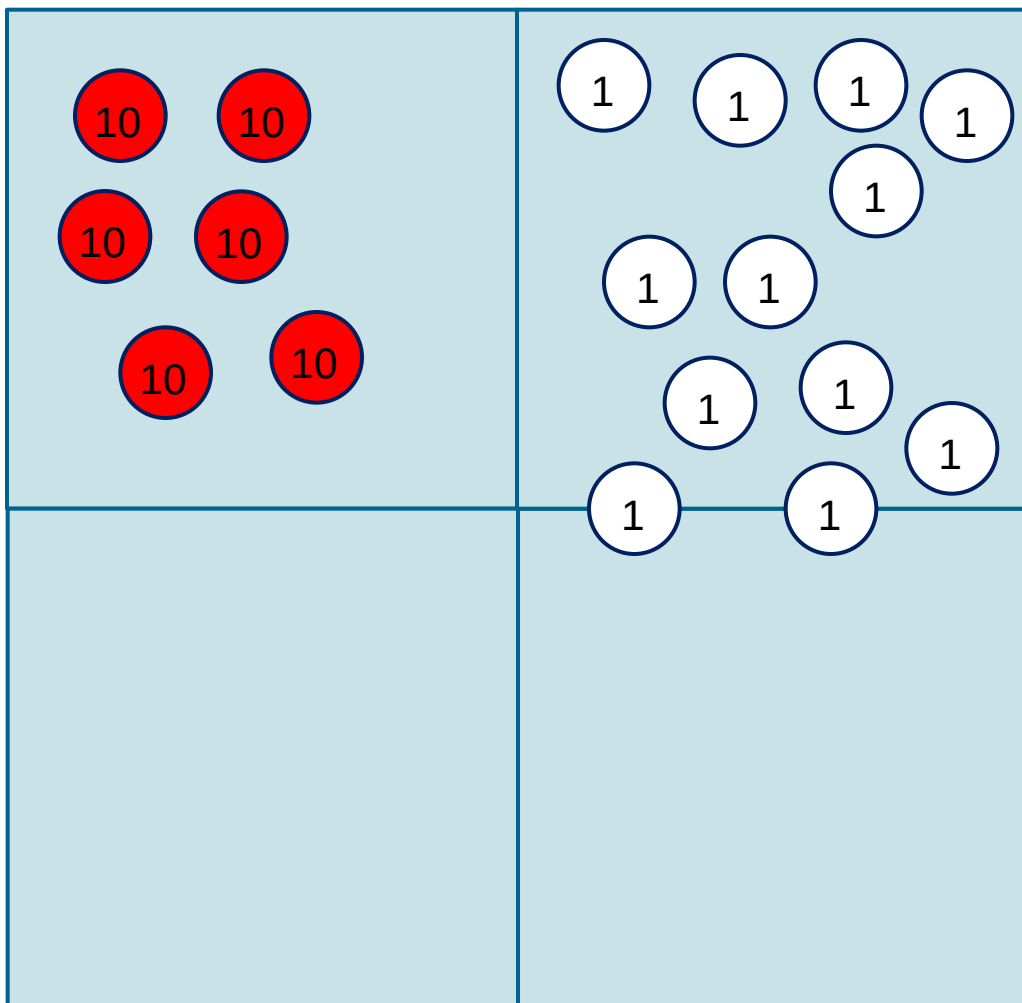


$$72 - 47 = 25$$

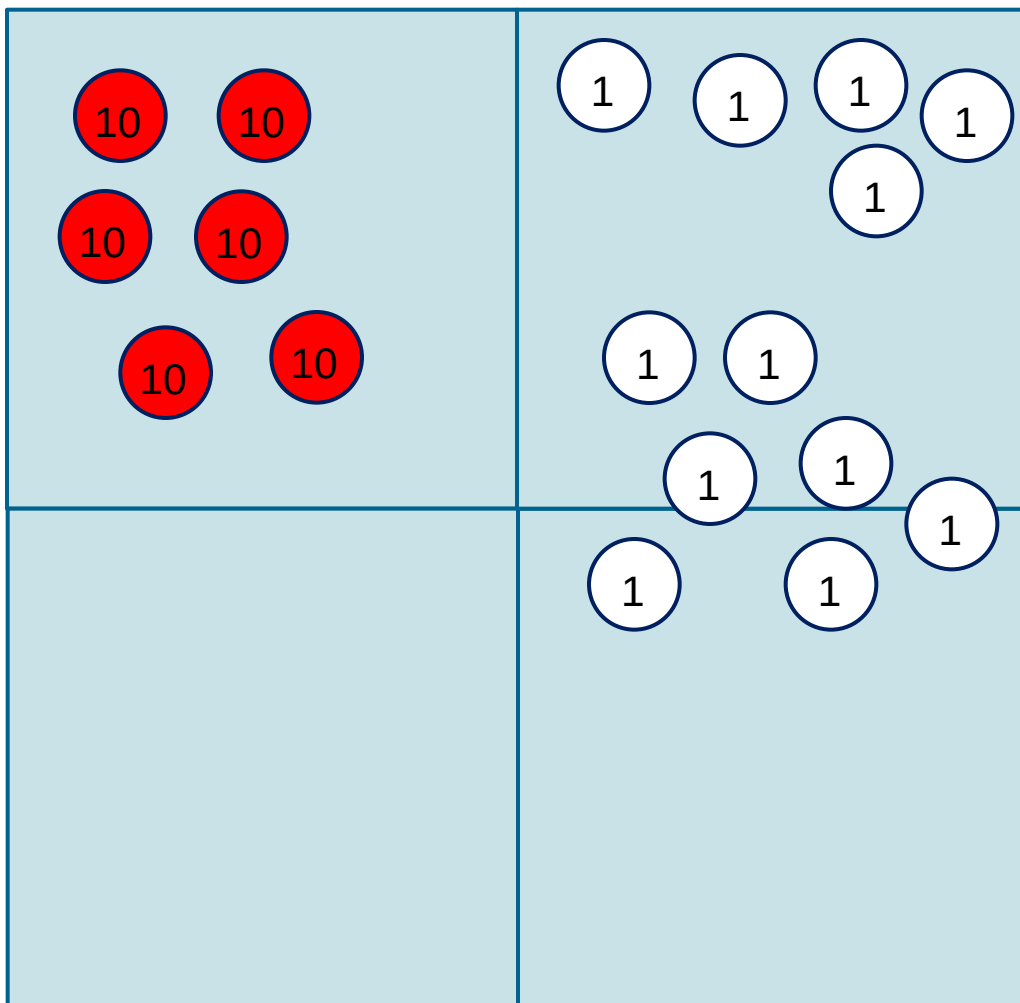




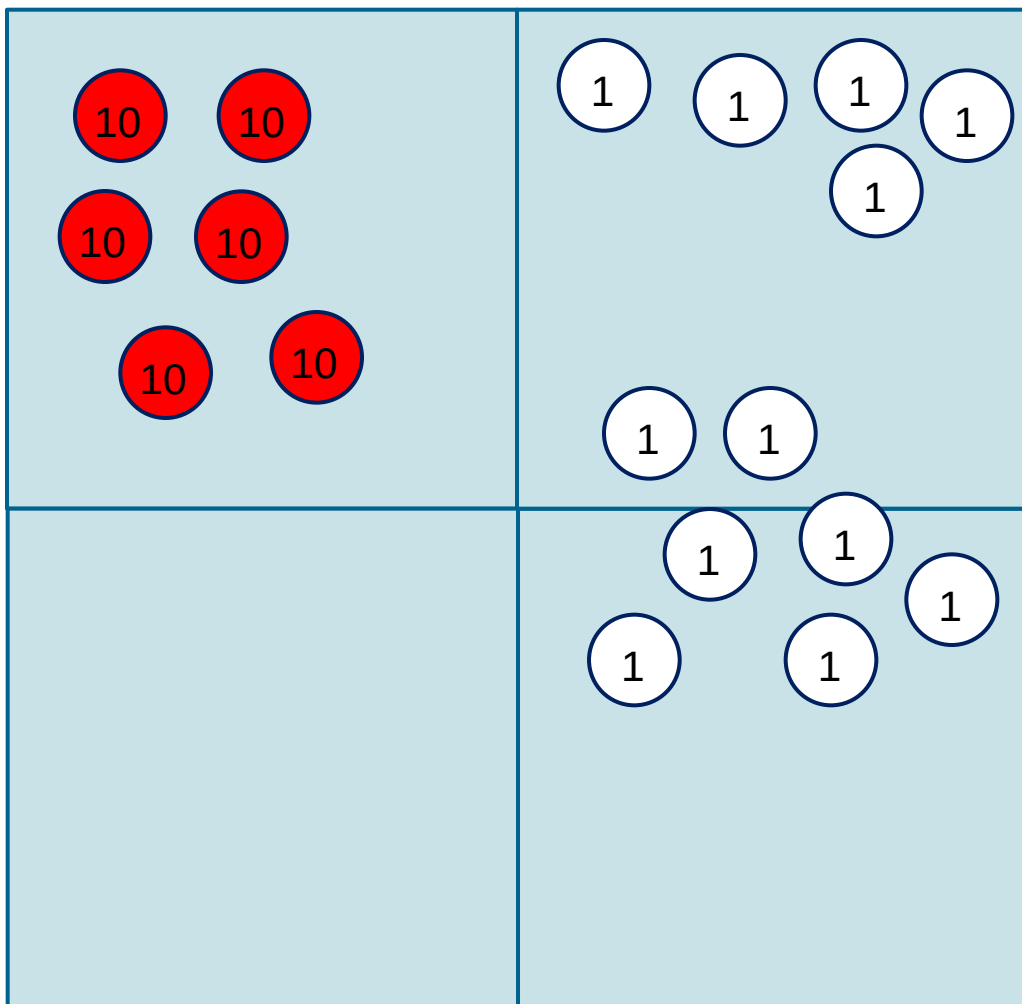
$$\begin{array}{r}
 \overset{6}{\cancel{7}}\overset{1}{2} \\
 - 47 \\
 \hline \\
 \hline
 \end{array}$$



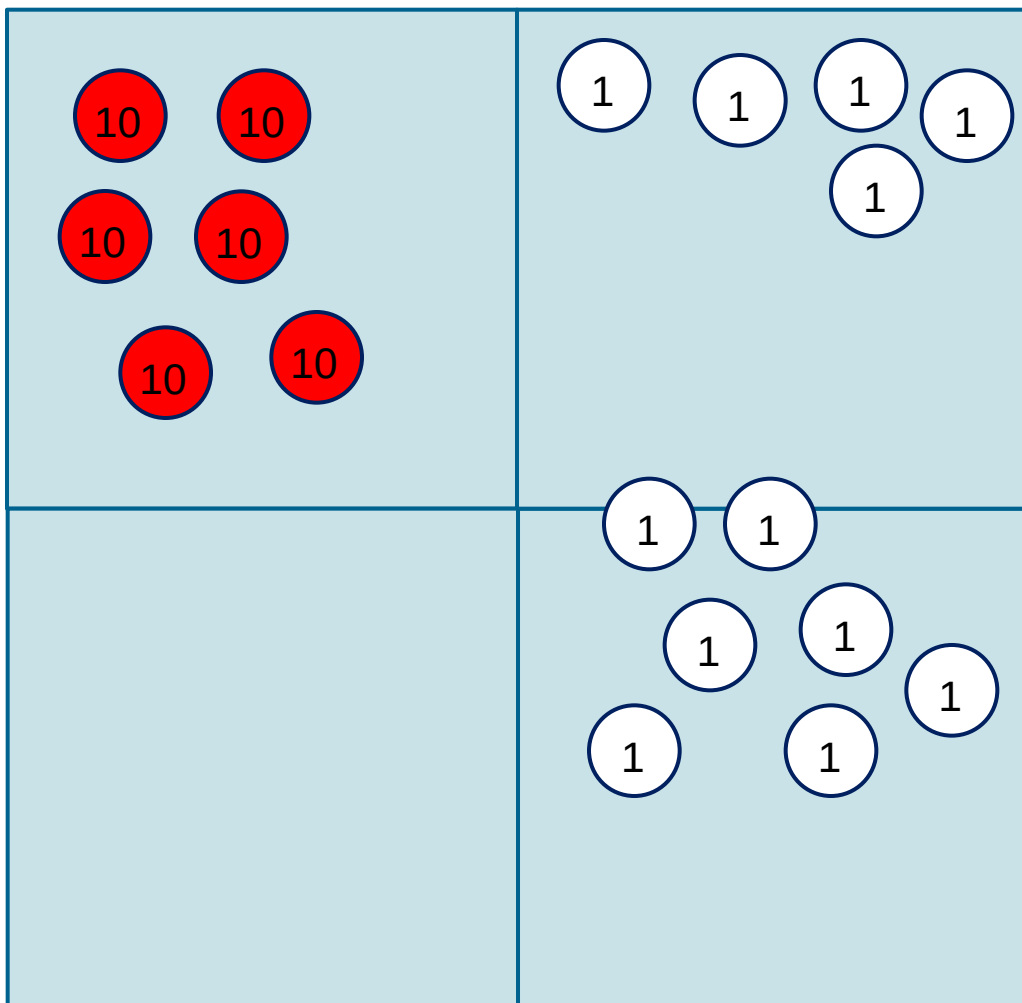
$$\begin{array}{r} \overset{6}{\cancel{7}} \overset{1}{2} \\ - 47 \\ \hline \\ \hline \end{array}$$



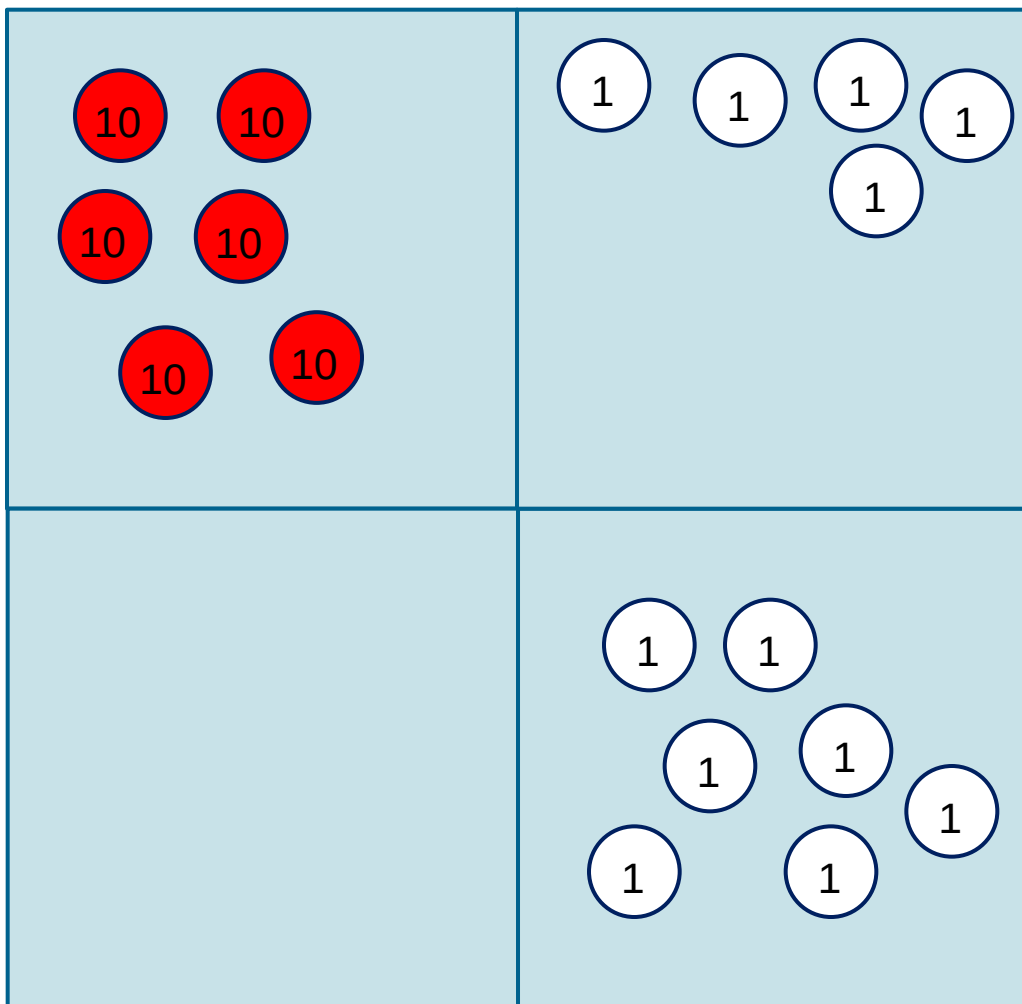
$$\begin{array}{r}
 \overset{6}{\cancel{7}}\overset{1}{2} \\
 - 47 \\
 \hline \\
 \hline
 \end{array}$$



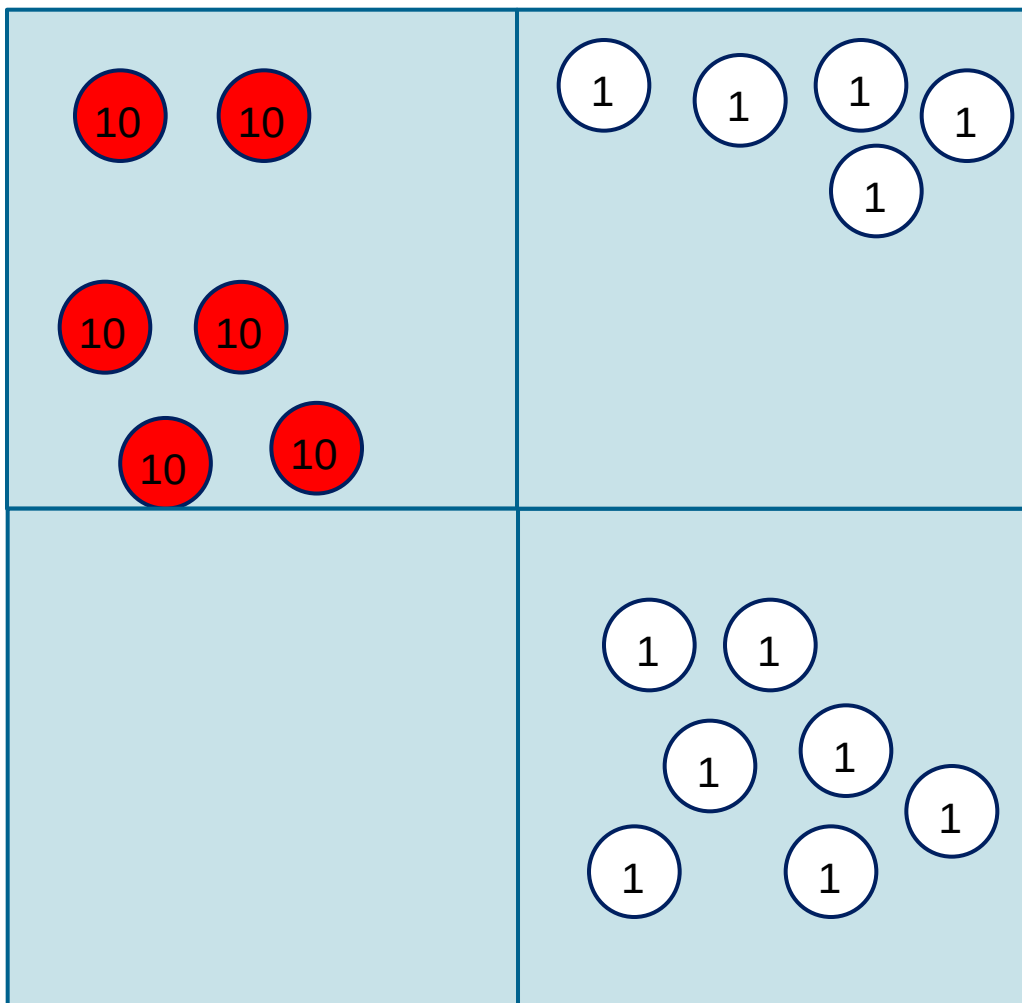
$$\begin{array}{r}
 \overset{6}{\cancel{7}} \overset{1}{2} \\
 - 47 \\
 \hline
 \hline
 \end{array}$$



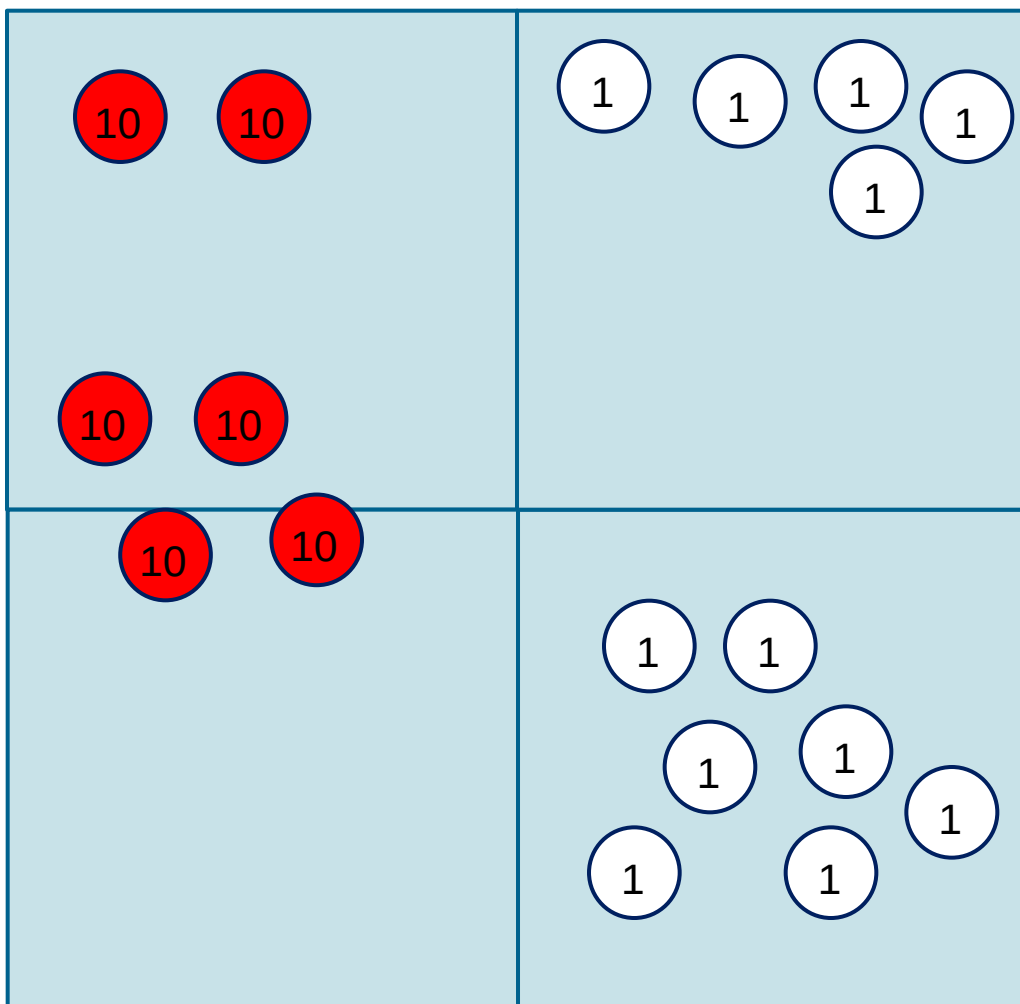
$$\begin{array}{r} \overset{6}{\cancel{7}} \overset{1}{2} \\ - 47 \\ \hline \\ \hline \end{array}$$



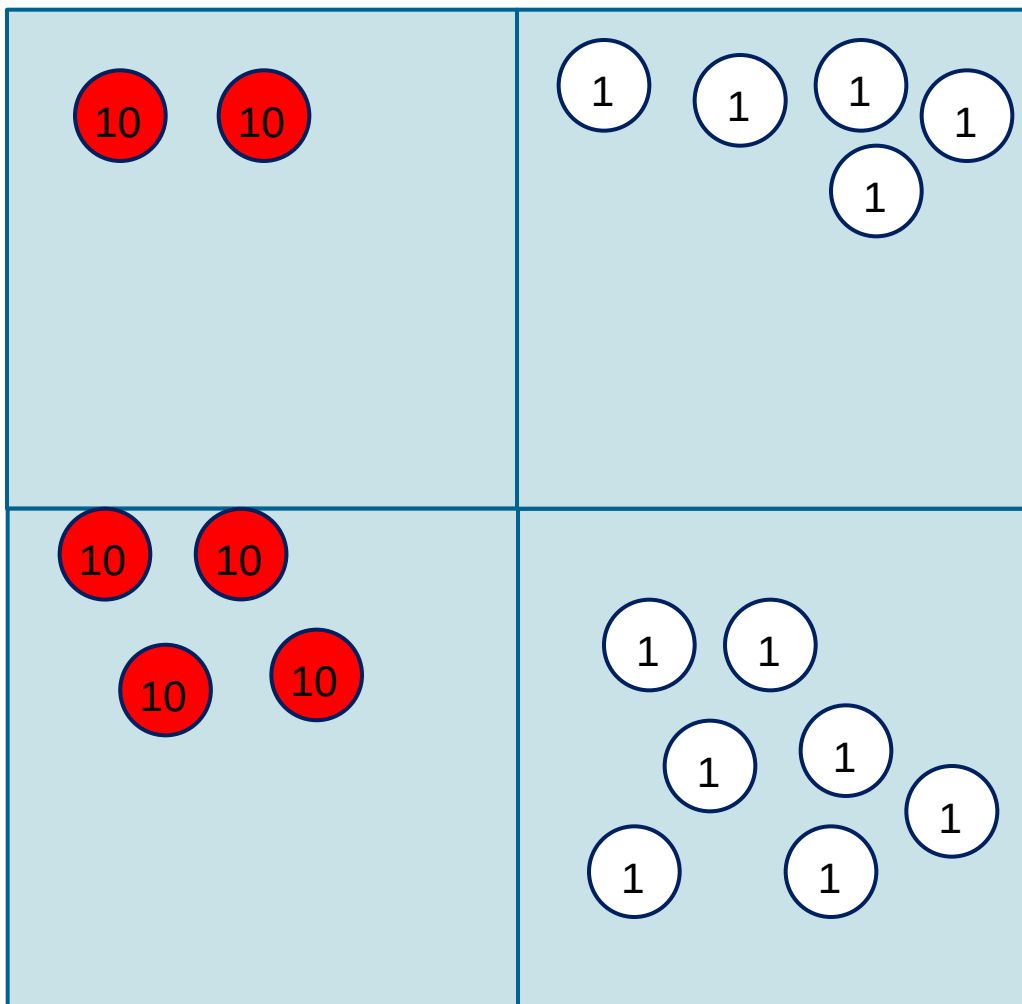
$$\begin{array}{r}
 \overset{6}{\cancel{7}} \overset{1}{2} \\
 - 47 \\
 \hline
 5
 \end{array}$$



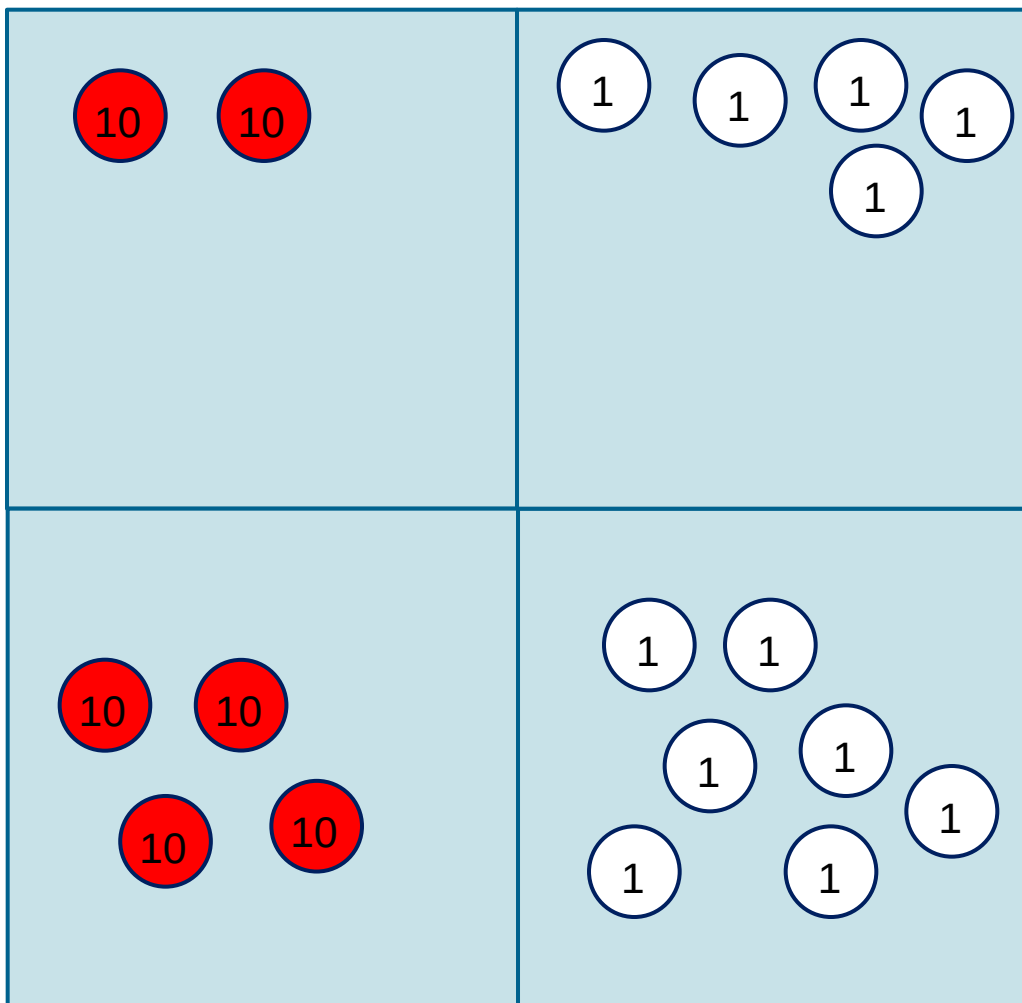
$$\begin{array}{r}
 \overset{6}{\cancel{7}}\overset{1}{2} \\
 - 47 \\
 \hline
 5
 \end{array}$$



$$\begin{array}{r}
 \overset{6}{\cancel{7}}\overset{1}{2} \\
 - 47 \\
 \hline
 5
 \end{array}$$



$$\begin{array}{r}
 \overset{6}{\cancel{7}} \overset{1}{2} \\
 - 47 \\
 \hline
 5
 \end{array}$$



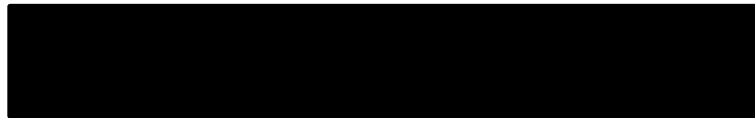
$$\begin{array}{r}
 \overset{6}{\cancel{7}}\overset{1}{2} \\
 - 47 \\
 \hline
 25
 \end{array}$$

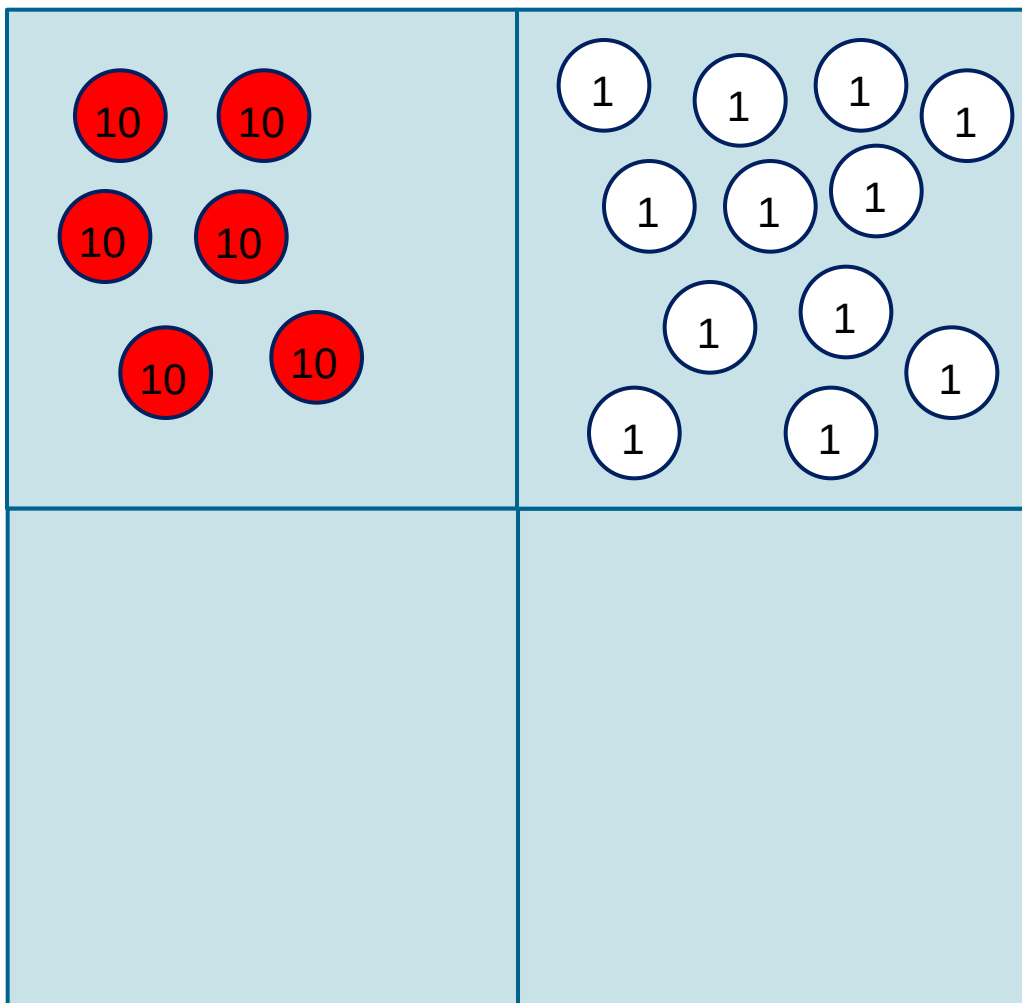
Addition and Subtraction: Both Ways

National Centre
for Excellence in the
Teaching of Mathematics

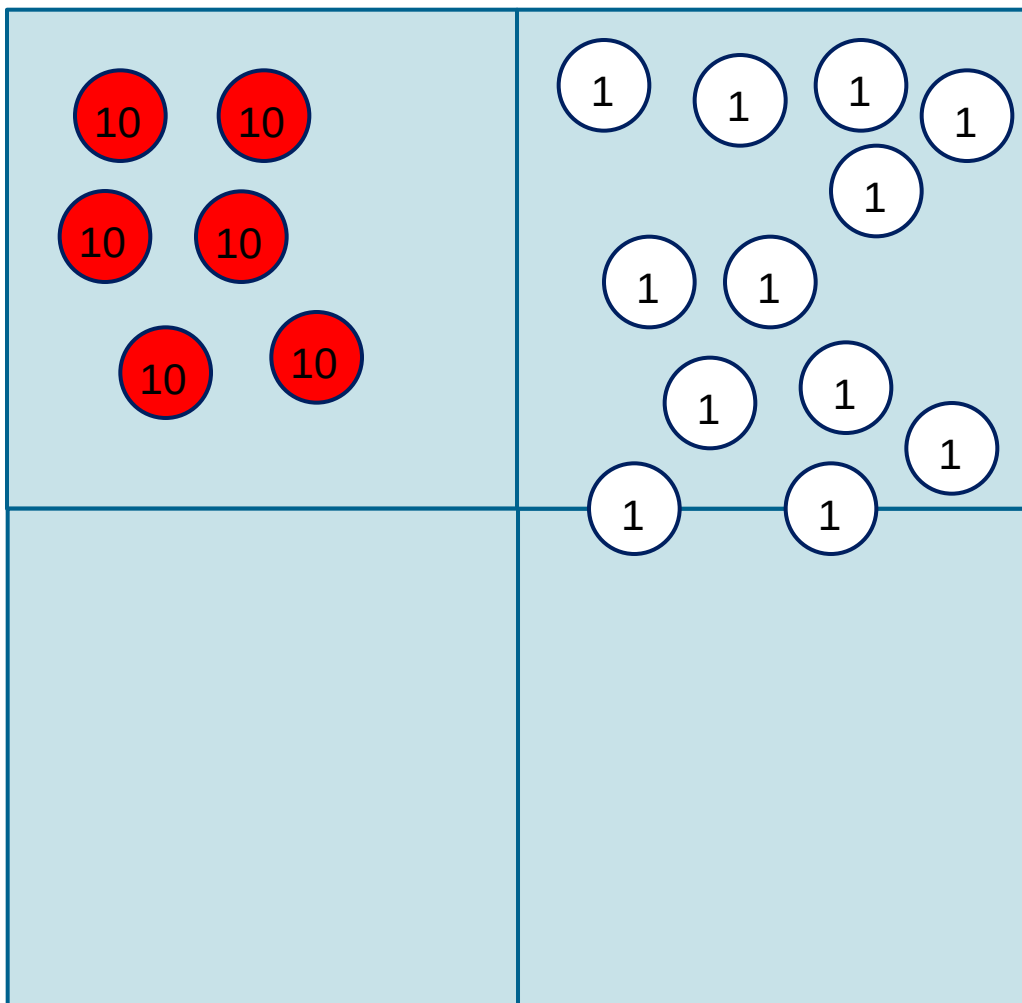


One or two relationships?

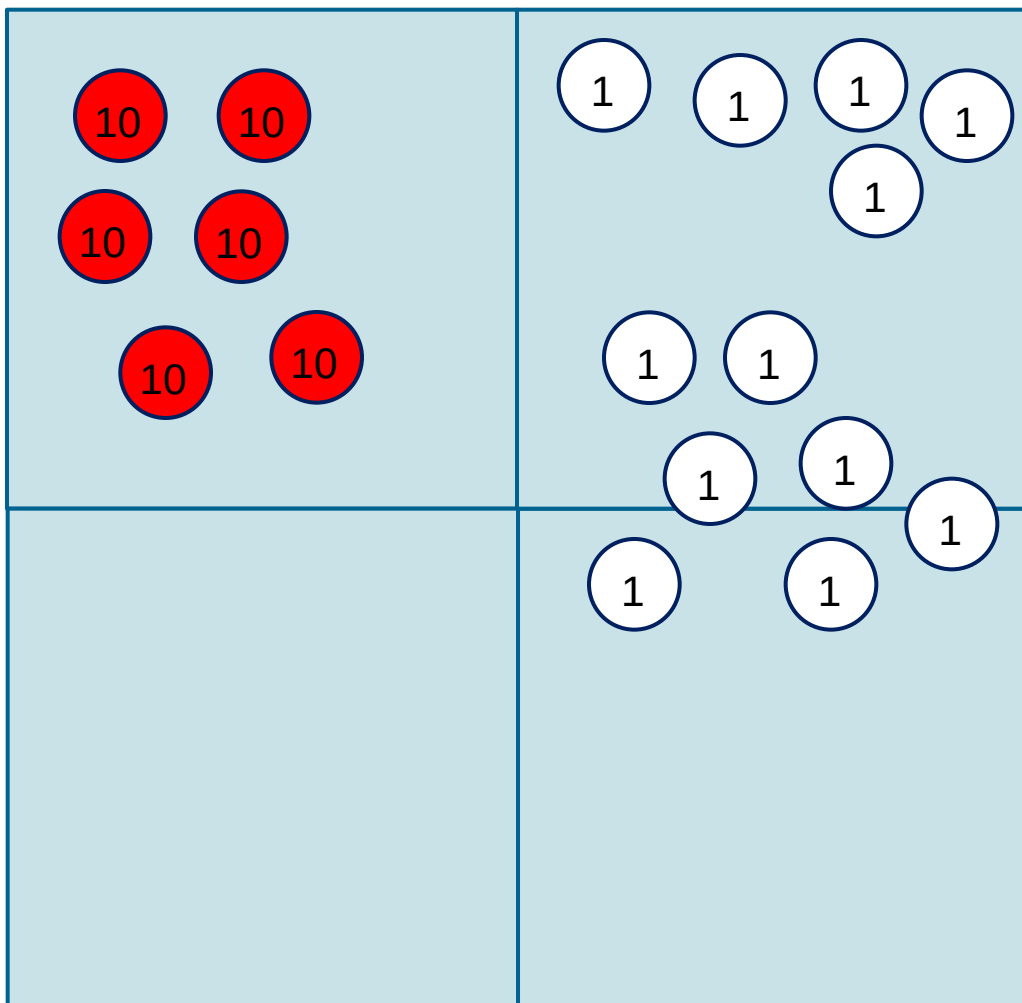




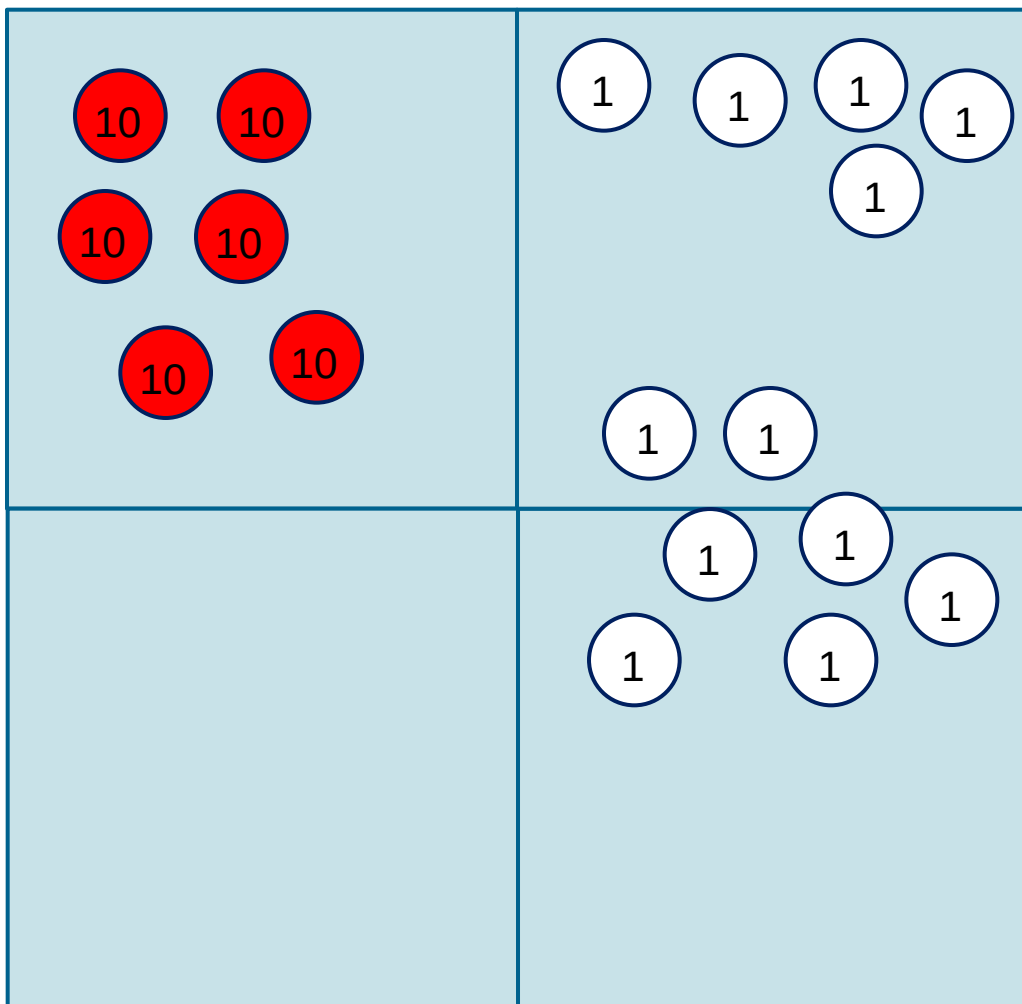
$$\begin{array}{r} \overset{6}{\cancel{7}} \overset{1}{2} \\ - 47 \\ \hline \\ \hline \end{array}$$



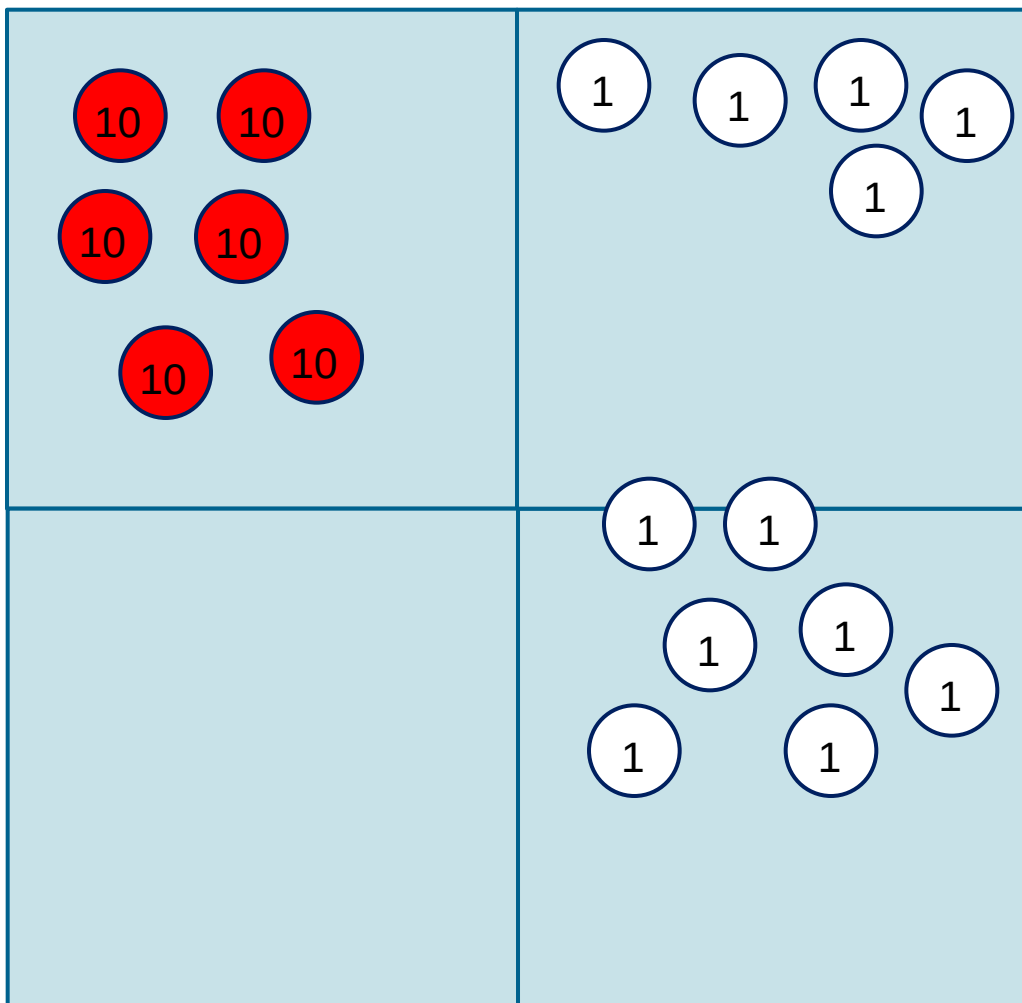
$$\begin{array}{r}
 \overset{6}{\cancel{7}}\overset{1}{2} \\
 - 47 \\
 \hline \\
 \hline
 \end{array}$$



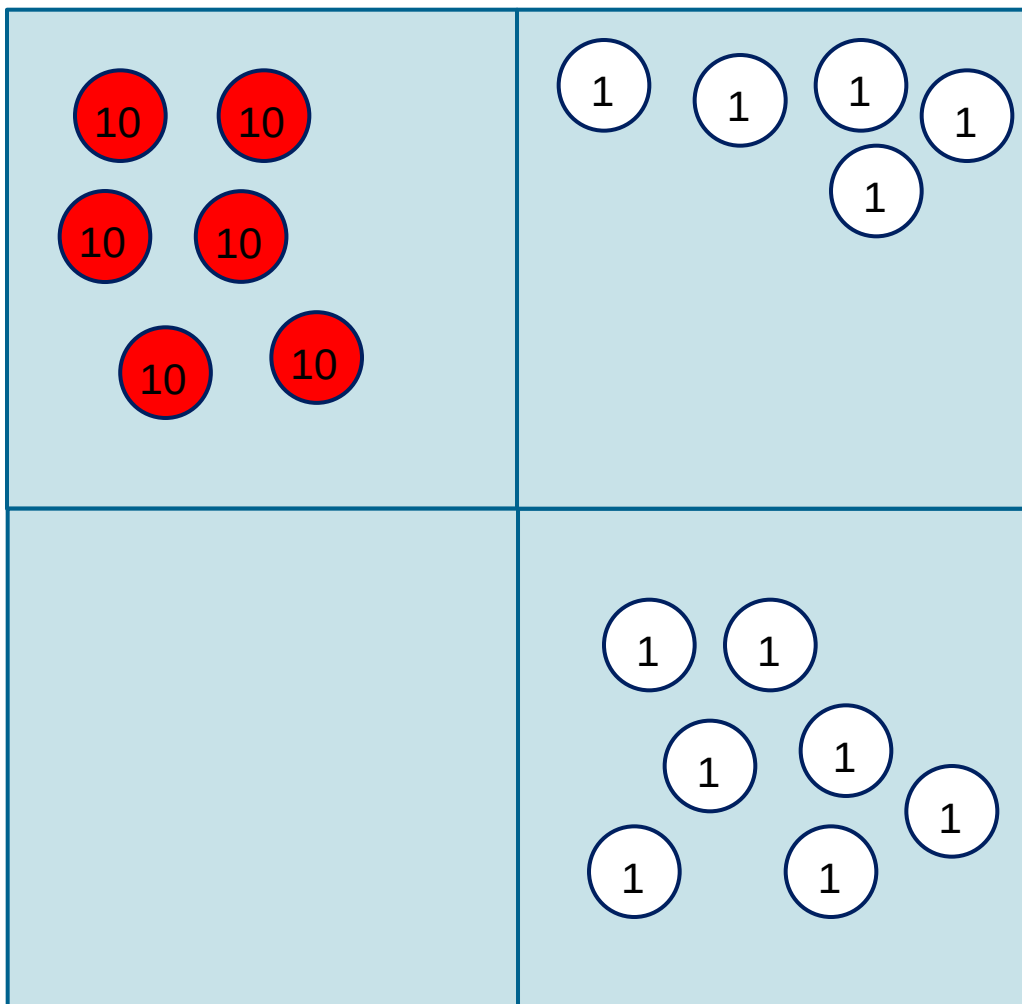
$$\begin{array}{r}
 \overset{6}{\cancel{7}}\overset{1}{2} \\
 - 47 \\
 \hline \\
 \hline
 \end{array}$$



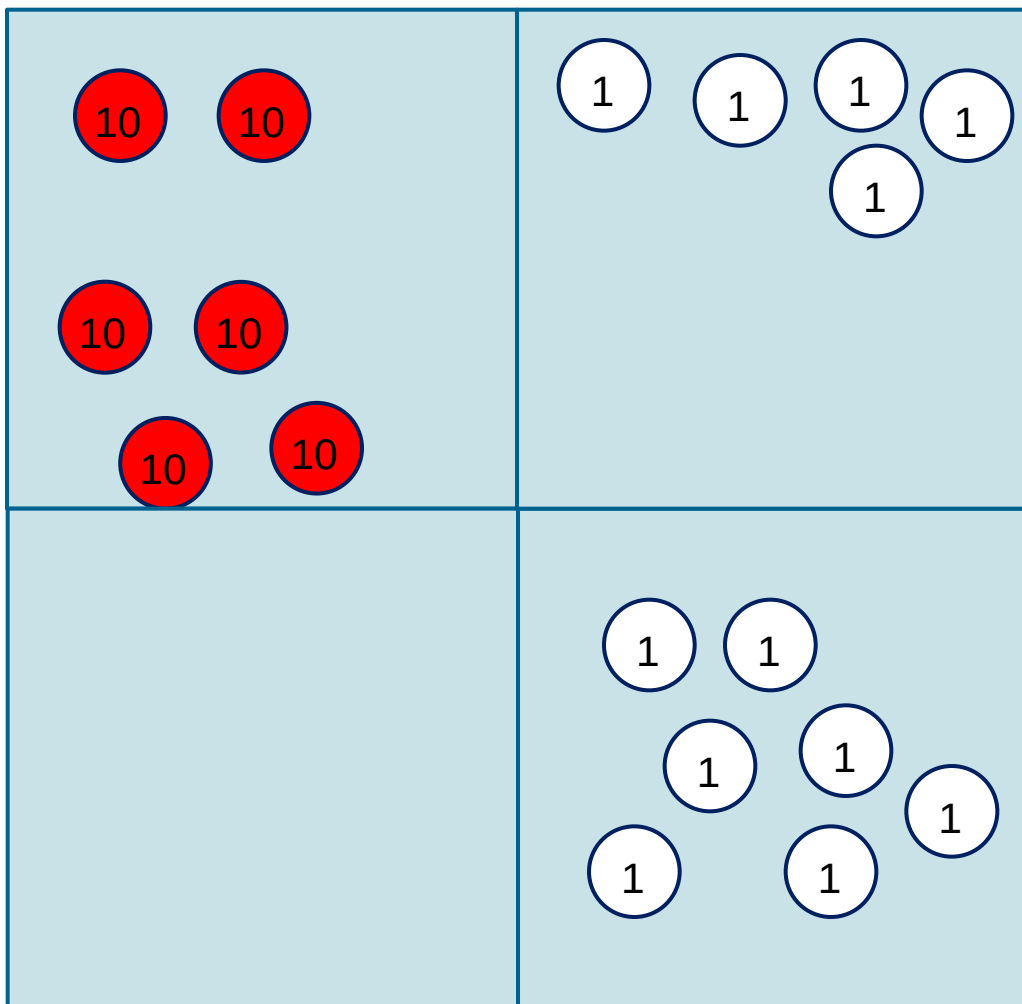
$$\begin{array}{r}
 \overset{6}{\cancel{7}} \overset{1}{2} \\
 - 47 \\
 \hline \\
 \hline
 \end{array}$$



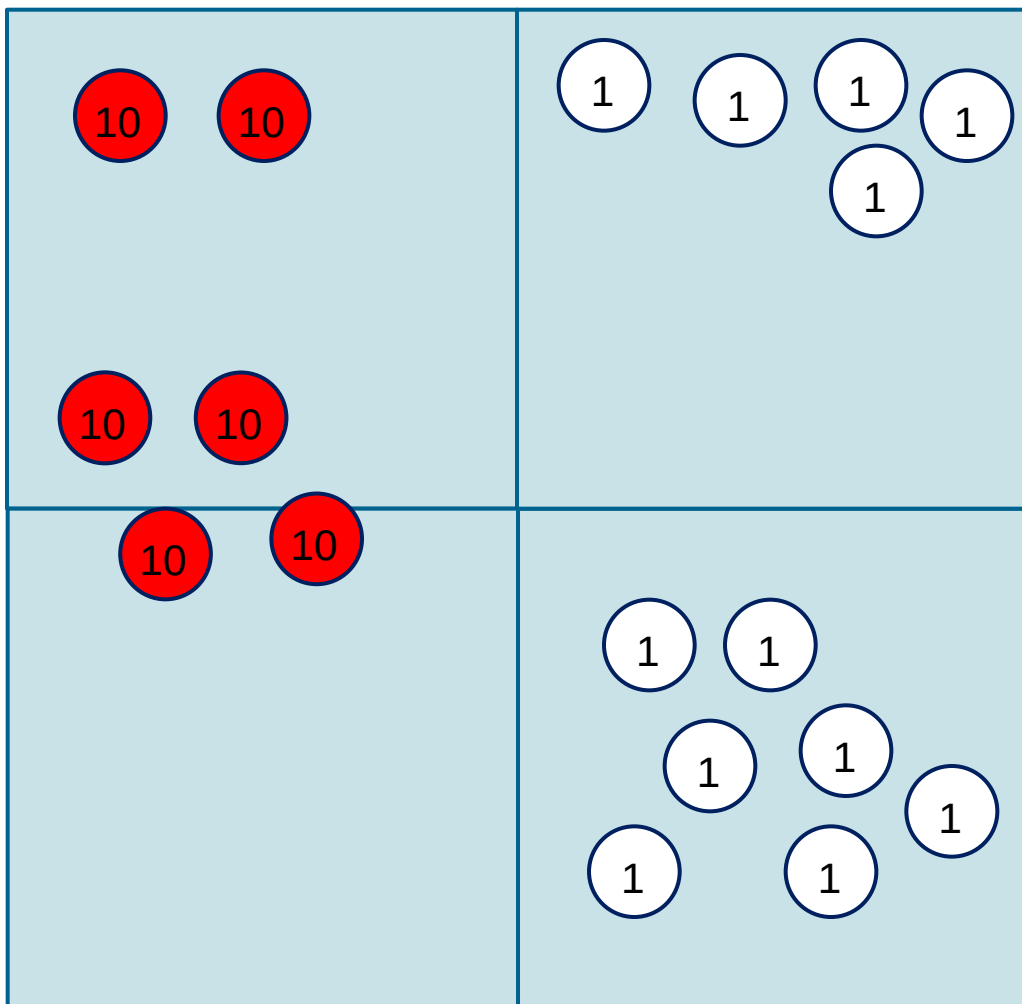
$$\begin{array}{r} \overset{6}{\cancel{7}} \overset{1}{2} \\ - 47 \\ \hline \\ \hline \end{array}$$



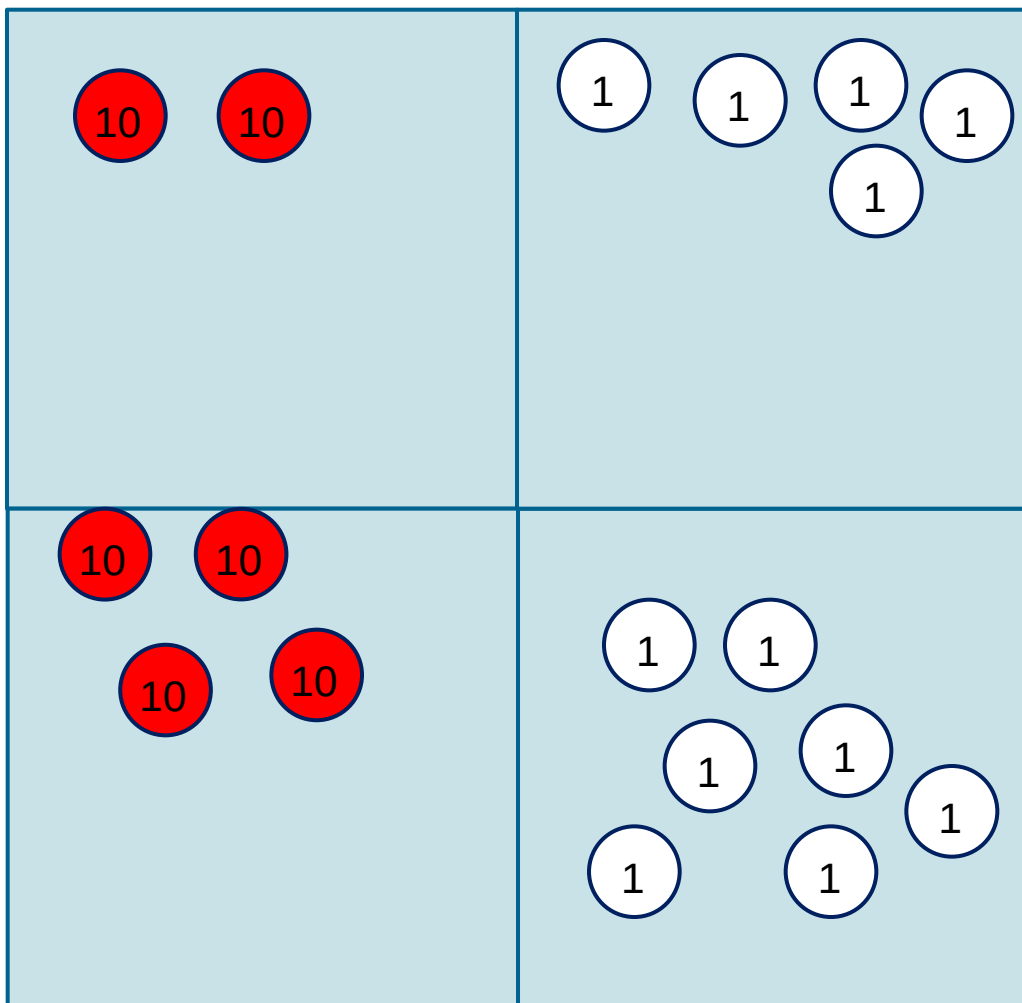
$$\begin{array}{r}
 \overset{6}{\cancel{7}} \overset{1}{2} \\
 - 47 \\
 \hline
 5
 \end{array}$$



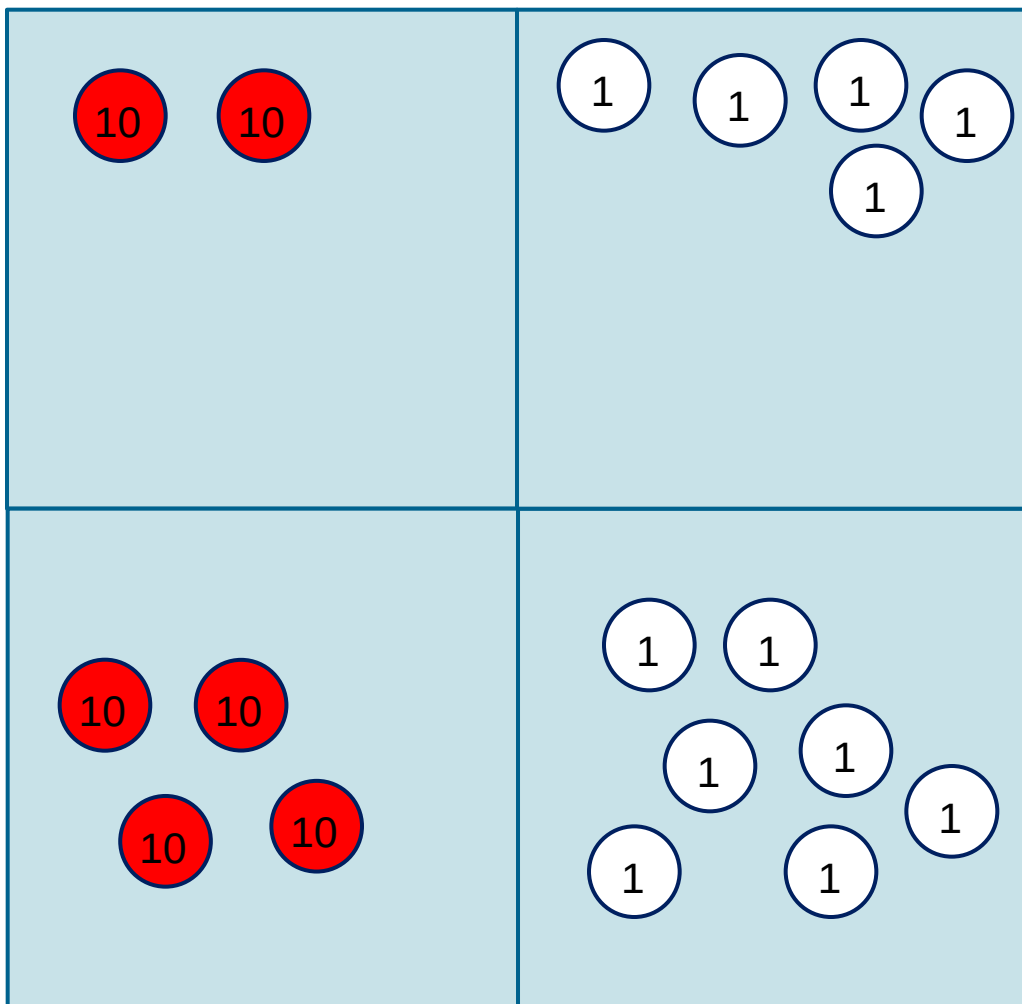
$$\begin{array}{r}
 \overset{6}{\cancel{7}}\overset{1}{2} \\
 - 47 \\
 \hline
 5
 \end{array}$$



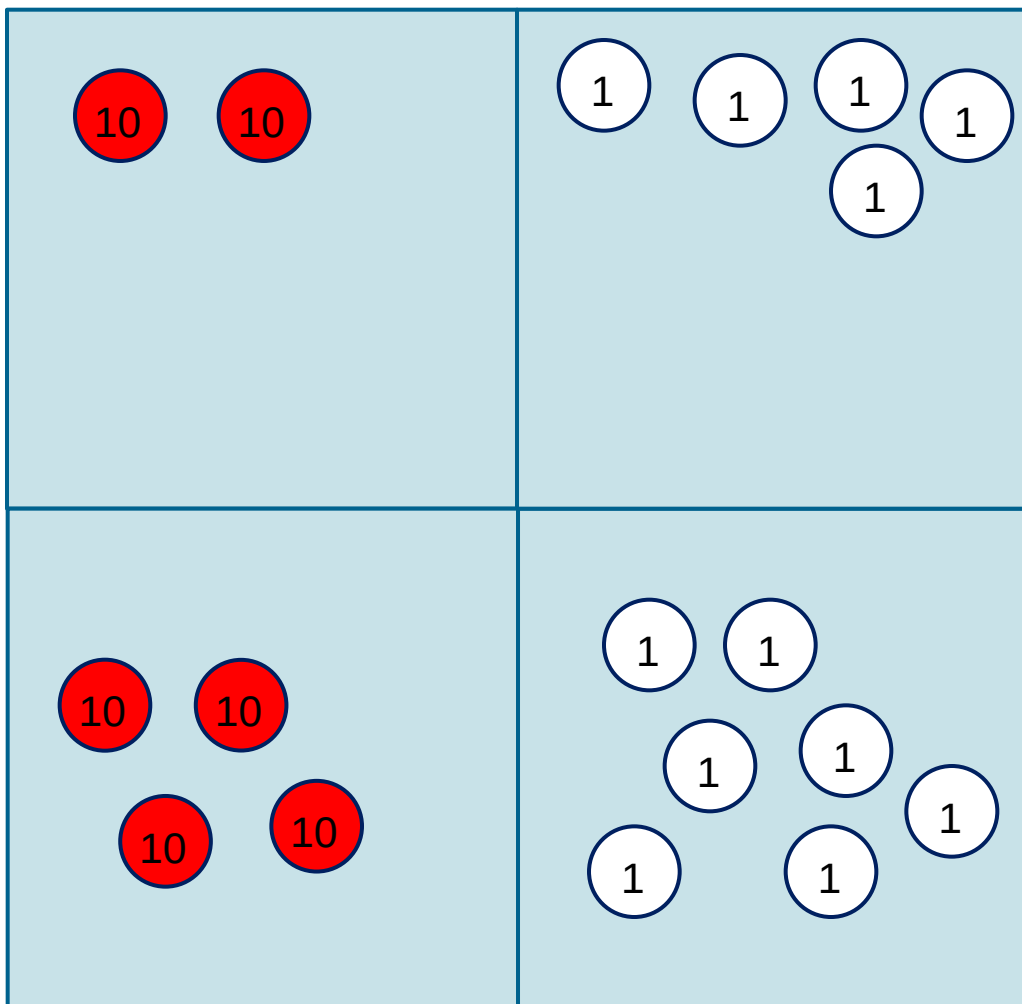
$$\begin{array}{r}
 \overset{6}{\cancel{7}}\overset{1}{2} \\
 - 47 \\
 \hline
 5
 \end{array}$$



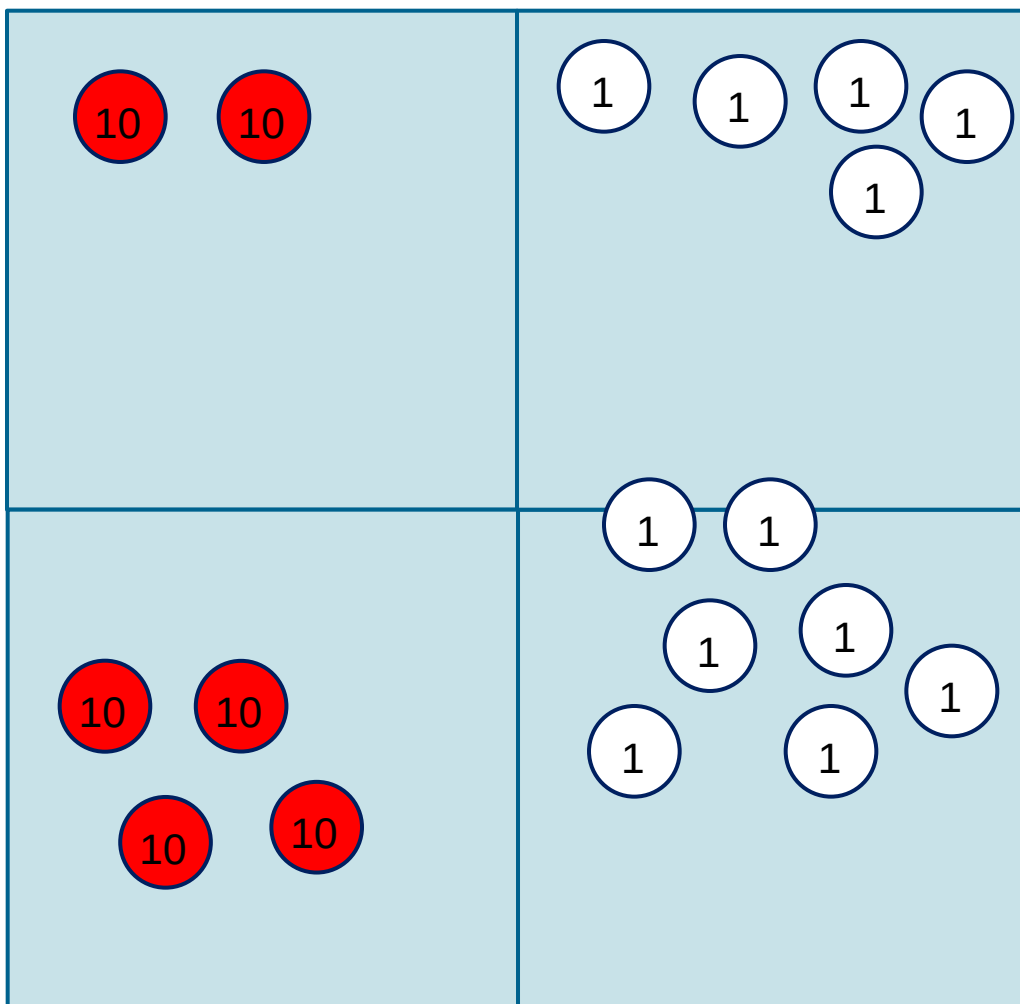
$$\begin{array}{r}
 \overset{6}{\cancel{7}} \overset{1}{2} \\
 - 47 \\
 \hline
 5
 \end{array}$$



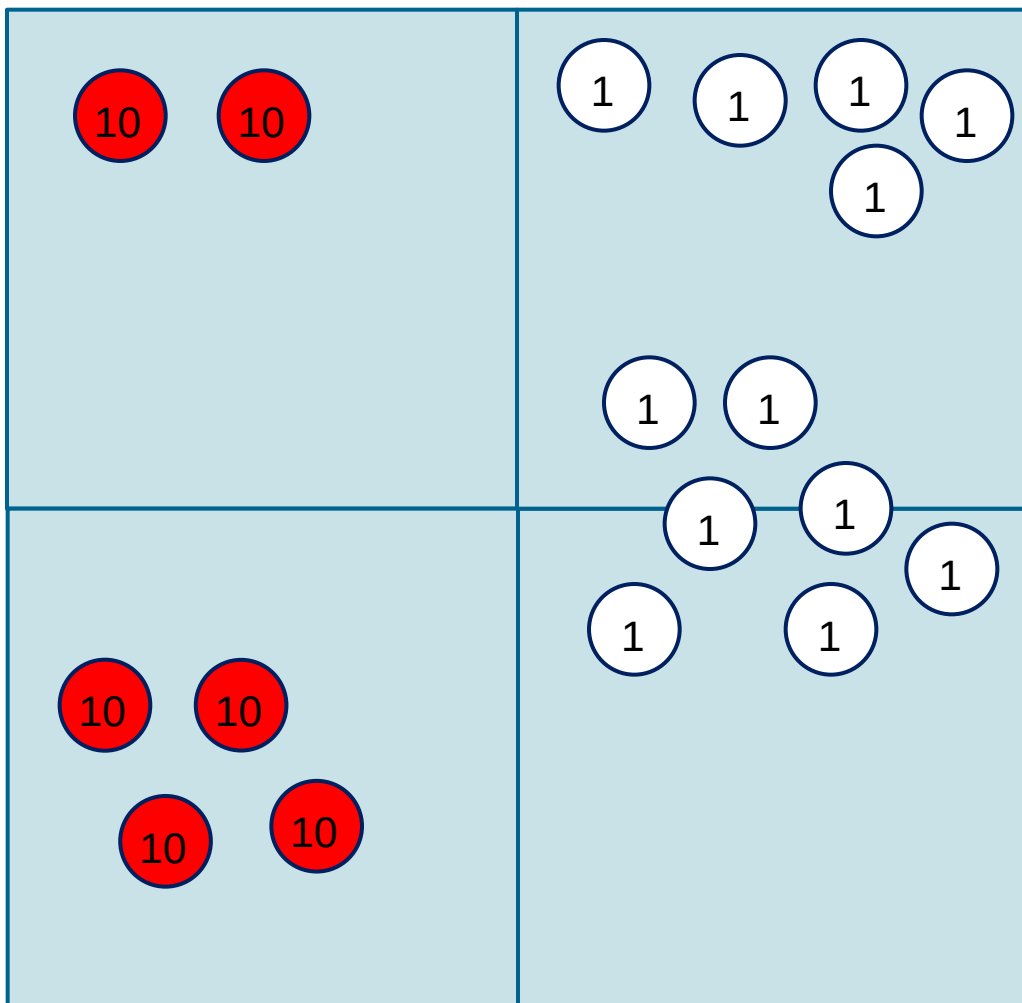
$$\begin{array}{r}
 \overset{6}{\cancel{7}}\overset{1}{2} \\
 - 47 \\
 \hline
 25
 \end{array}$$



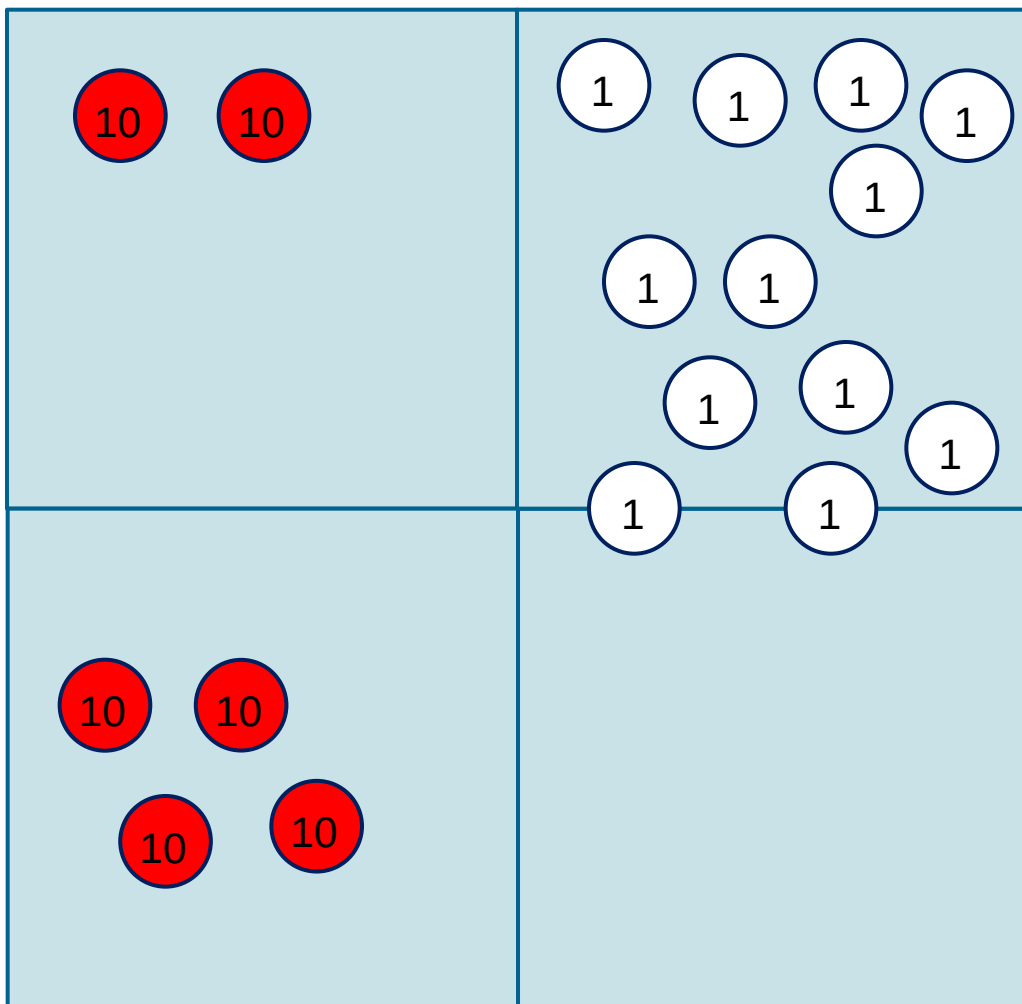
$$\begin{array}{r} 47 \\ + 25 \\ \hline \\ \hline \end{array}$$



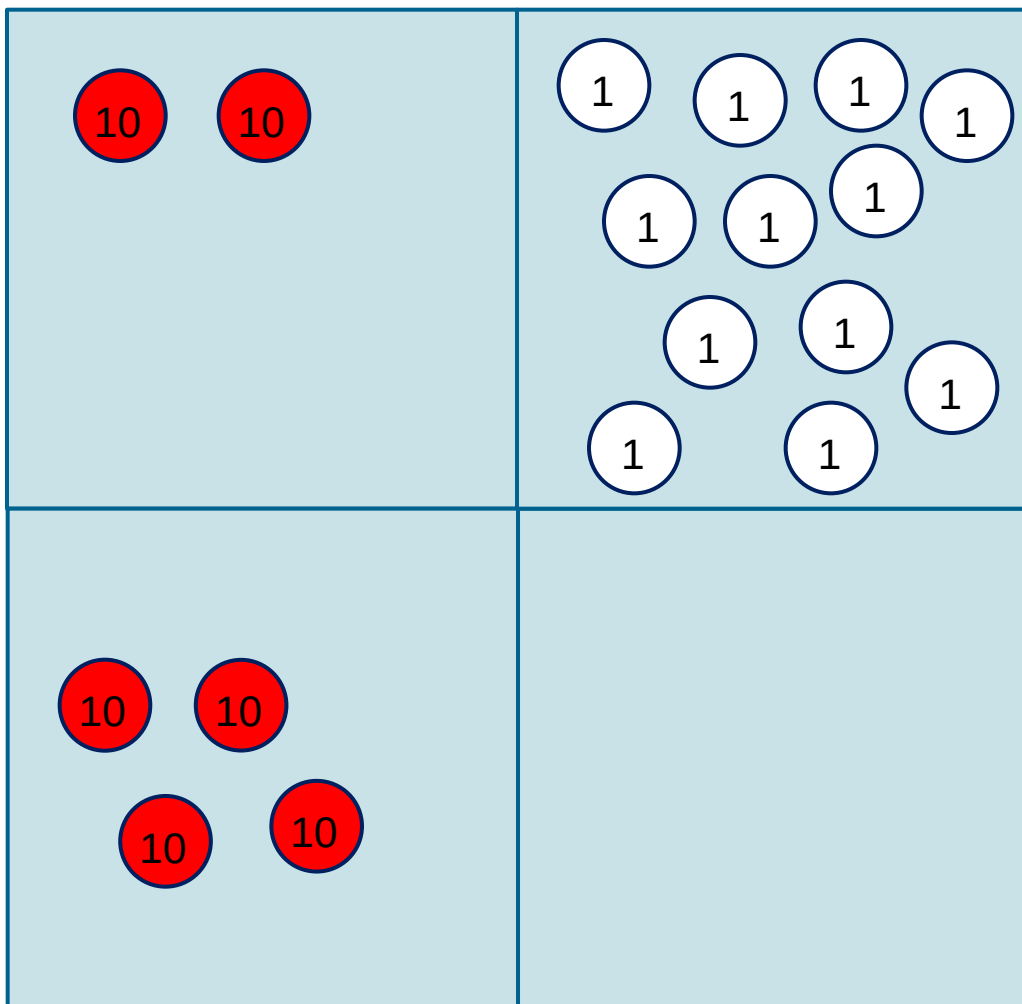
$$\begin{array}{r} 47 \\ + 25 \\ \hline \\ \hline \end{array}$$



$$\begin{array}{r} 47 \\ + 25 \\ \hline \hline \end{array}$$

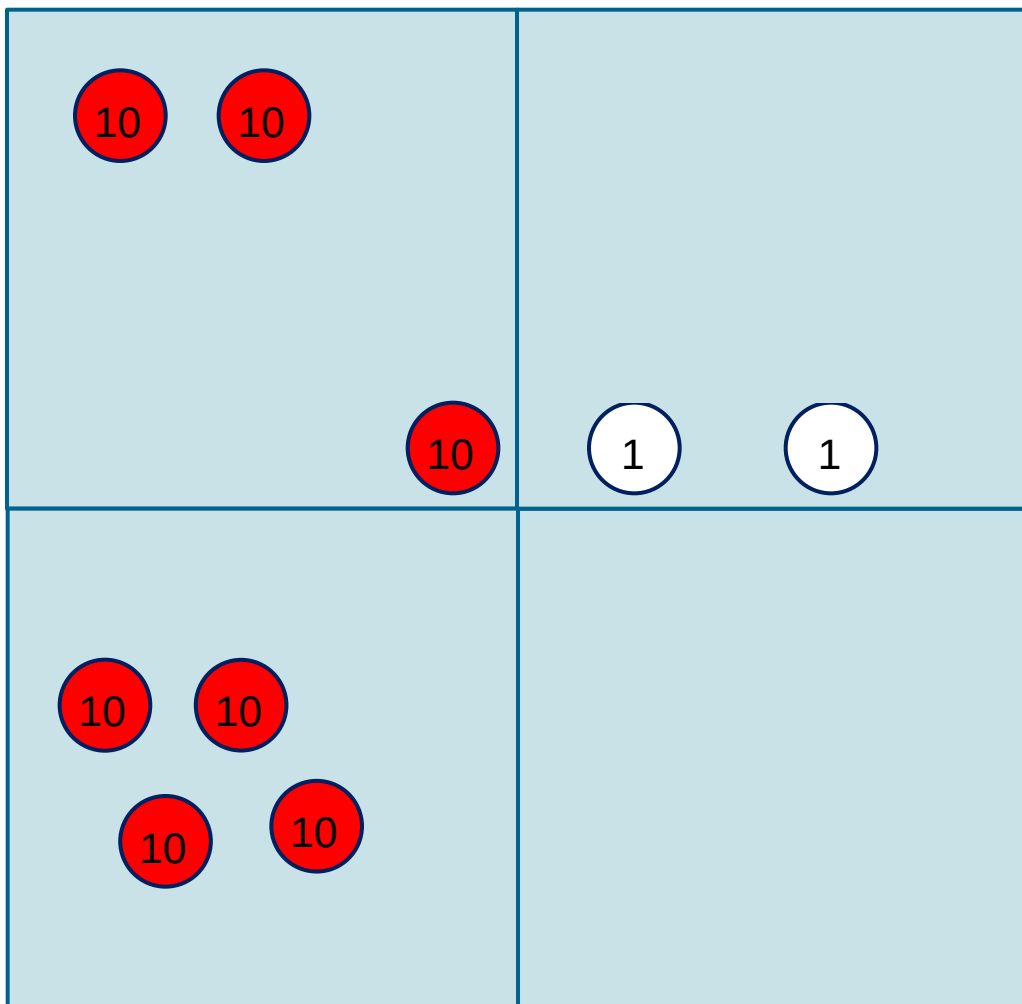


$$\begin{array}{r} 47 \\ + 25 \\ \hline \\ \hline \end{array}$$

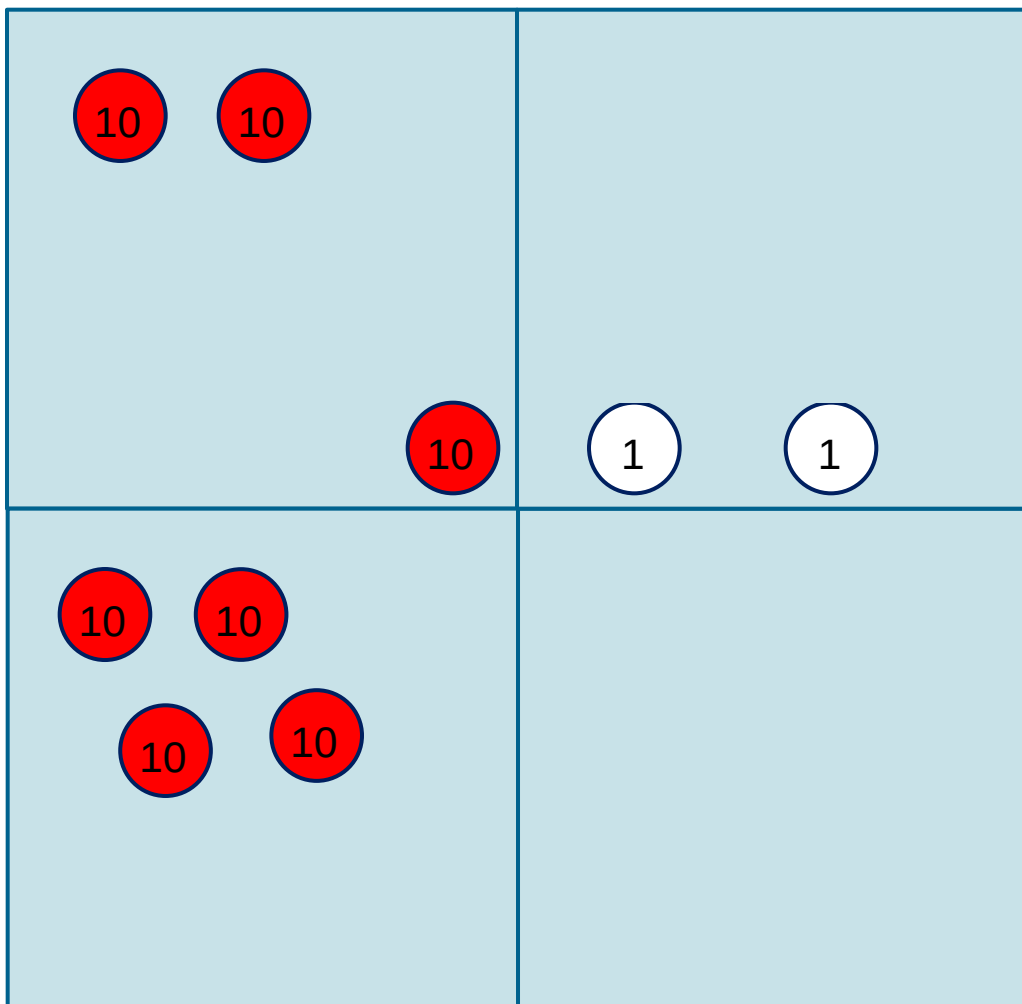


$$\begin{array}{r} 47 \\ +25 \\ \hline \end{array}$$

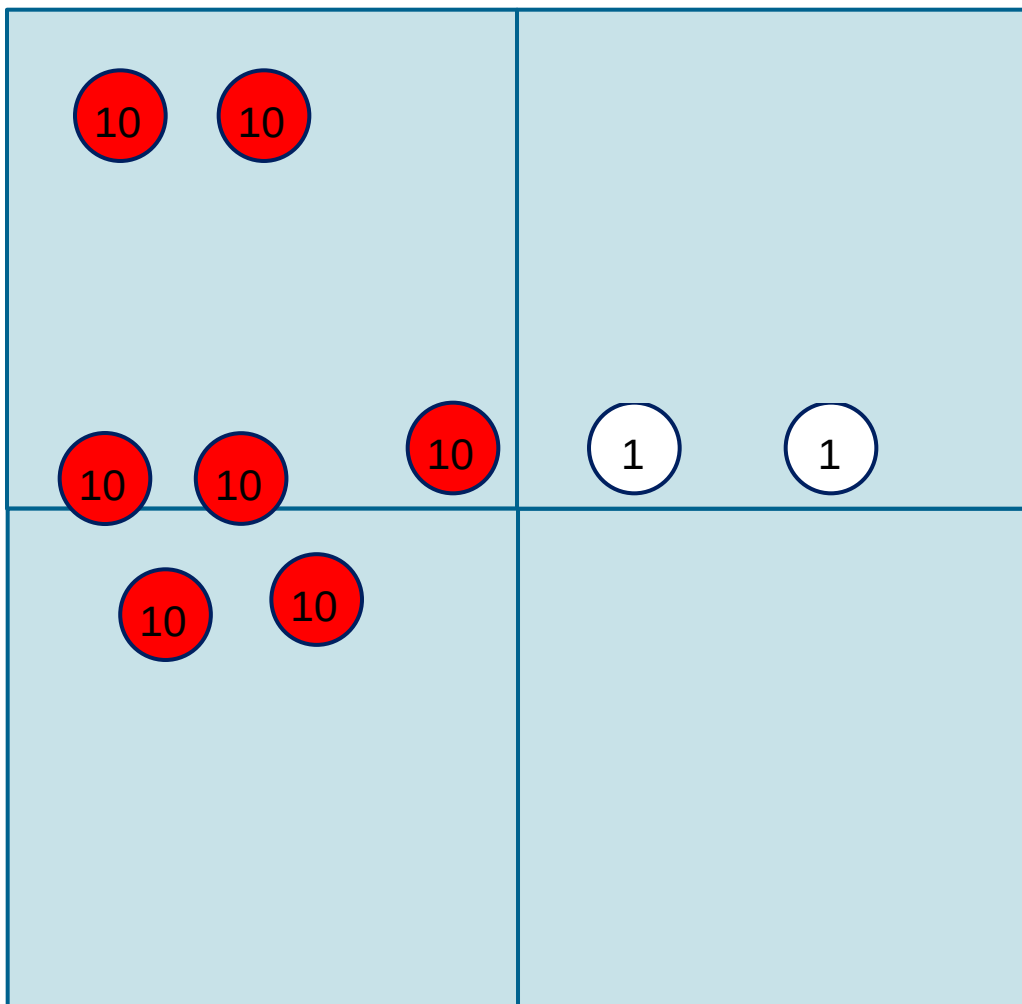
12



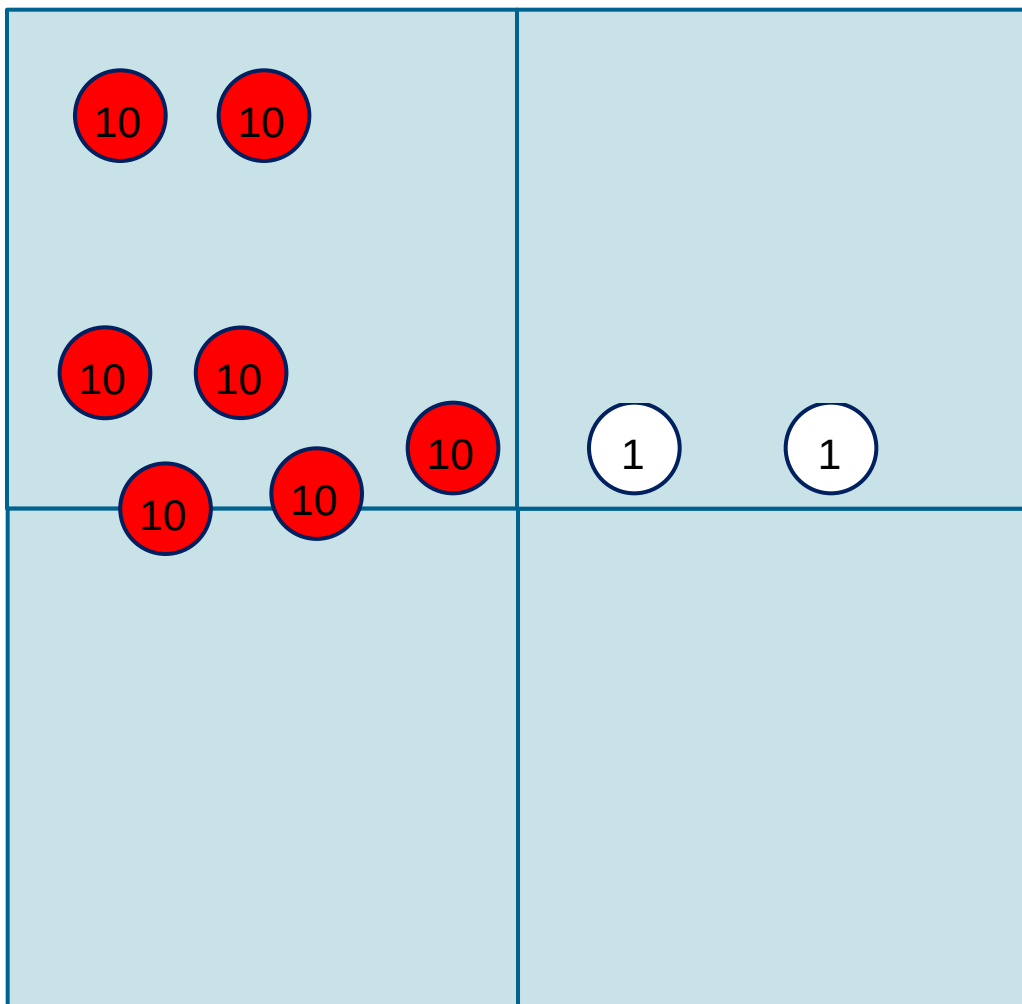
$$\begin{array}{r} 47 \\ +25 \\ \hline 2 \\ \hline 1 \end{array}$$



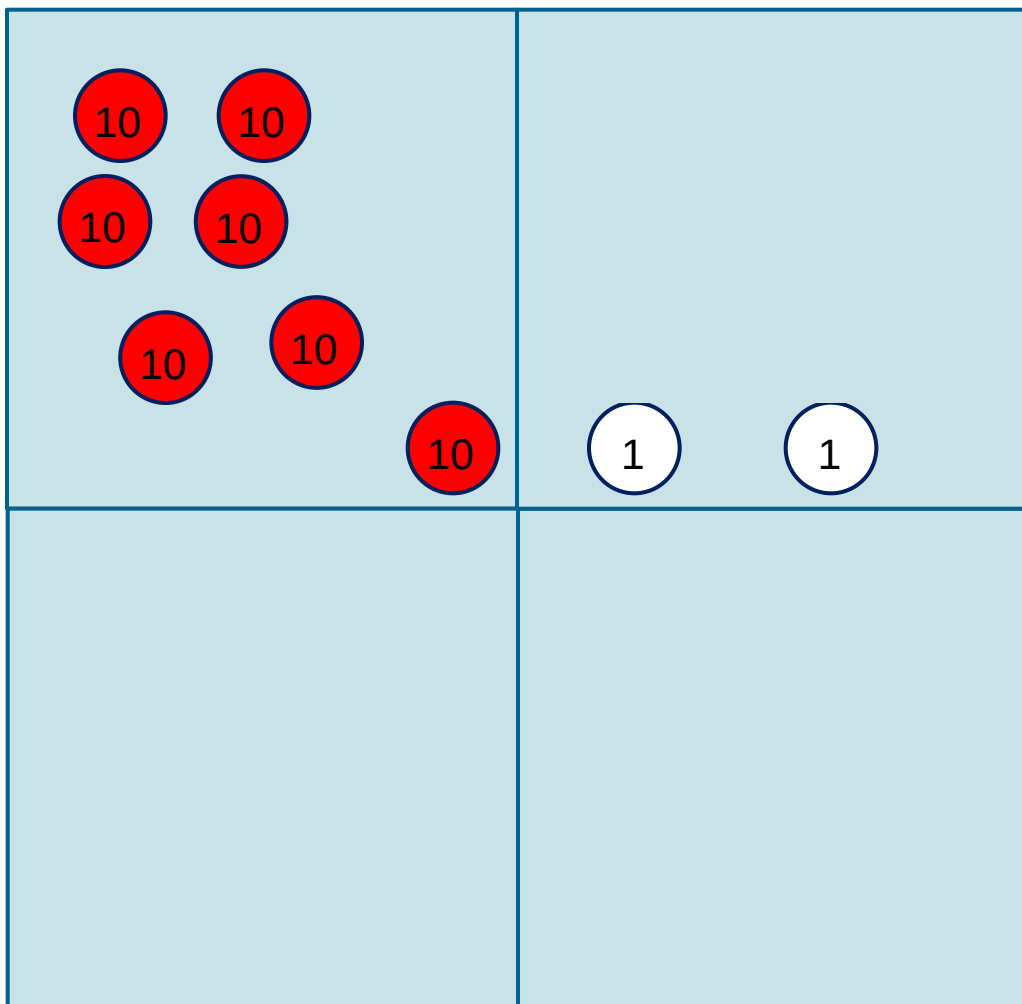
$$\begin{array}{r} 47 \\ +25 \\ \hline 2 \\ \hline 1 \end{array}$$



$$\begin{array}{r} 47 \\ +25 \\ \hline 2 \\ \hline 1 \end{array}$$



$$\begin{array}{r} 47 \\ +25 \\ \hline 2 \\ \hline 1 \end{array}$$



$$\begin{array}{r} 47 \\ +25 \\ \hline 72 \\ \hline 1 \end{array}$$

Multiplication

Progression in Teaching Multiplication

Mental Skills

Recognise the size and position of numbers

Count on in different steps 2s, 5s, 10s

Double numbers up to 10

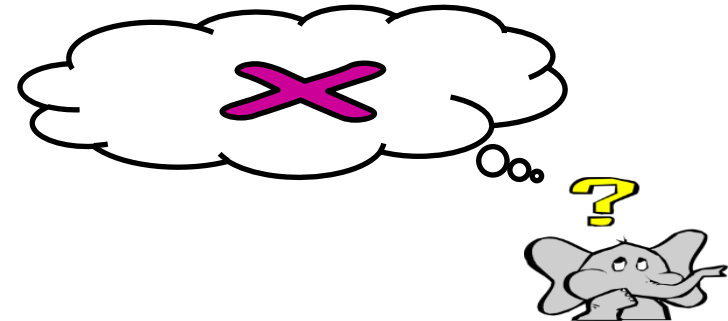
Recognise multiplication as repeated addition

Quick recall of multiplication facts

Use known facts to derive associated facts

Multiplying by 10, 100, 1000 and understanding the effect

Multiplying by multiples of 10



Models and Images

Counting apparatus

Place value apparatus

Arrays

100 squares

Number tracks

Numbered number lines

Marked but unnumbered lines

Empty number lines.

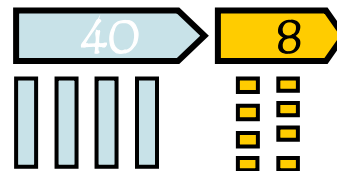
Multiplication squares

Counting stick

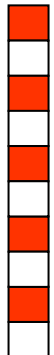
Bead strings

Models and Images charts

ITPs - Multiplication grid, Number Dials, Multiplication Facts



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Vocabulary

lots of

groups of

times

multiply

multiplication

multiple

product

once, twice, three times

array, row, column

double

repeated addition

multiplication **product**
once, twice, three times
double **groups of**
repeated addition **lots of**
array, row, column **multiply**
times **multiple**

Children need to understand the concept of multiplication, that it is:

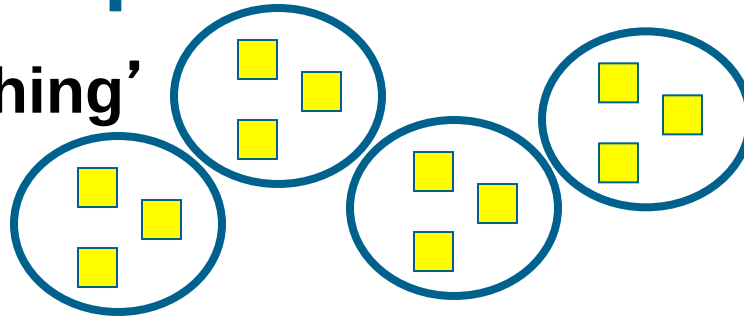
- **Repeated addition**
- **Can be represented as an array**

They also need to understand and work with certain principles:

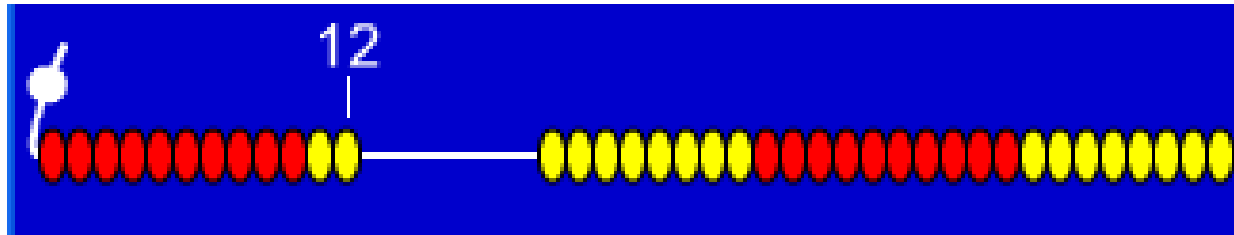
- **Inverse of division**
- **Is commutative i.e. $3 \times 5 = 5 \times 3$**
- **Is associative i.e. $2 \times (3 \times 5) = (2 \times 3) \times 5$**

Models for multiplication

Lots of the 'same thing'



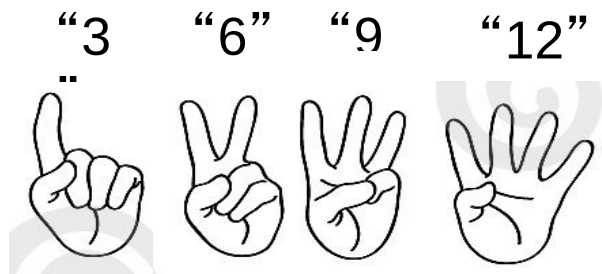
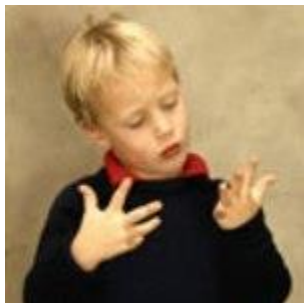
Bead Bar



Number Line

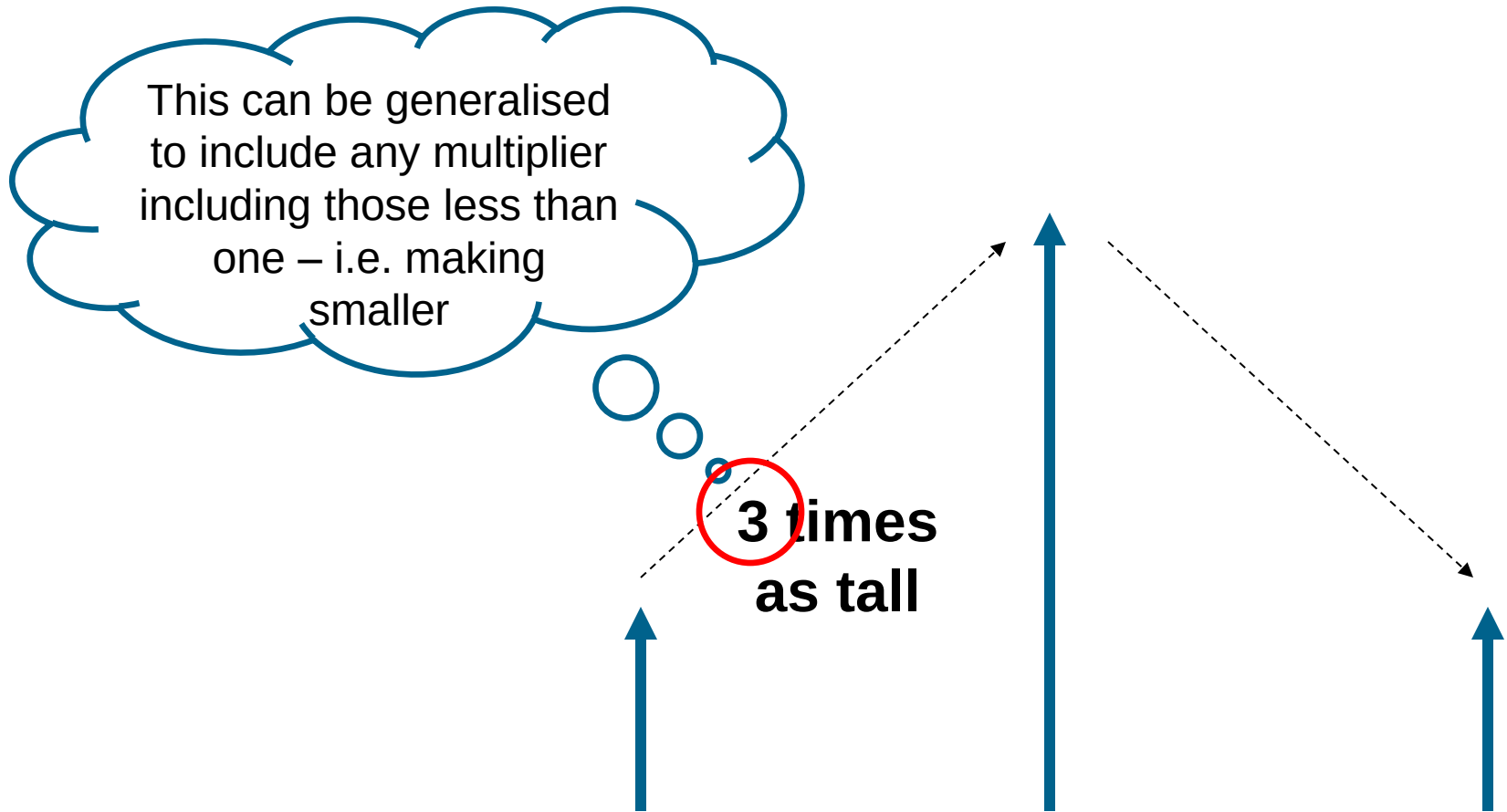


Fingers

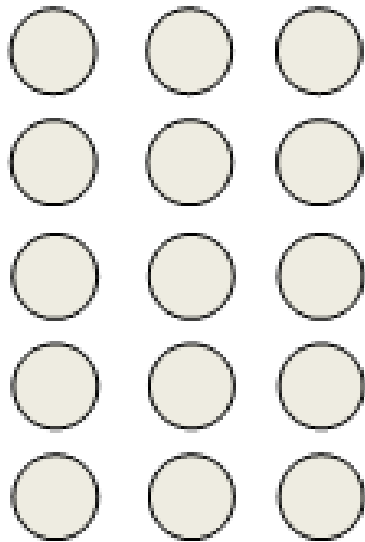


Models for multiplication

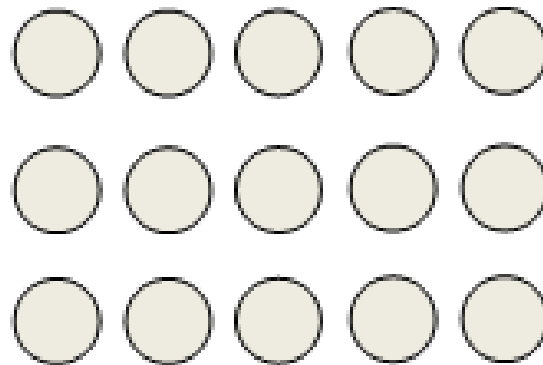
Scaling



Children need to understand how arrays link to multiplication through repeated addition and be able to create their own arrays.



$$3 + 3 + 3 + 3 + 3 = 15$$



$$5 + 5 + 5 = 15$$

Creating arrays on squared paper (this also links to understanding area).

$$4 \times 7 =$$

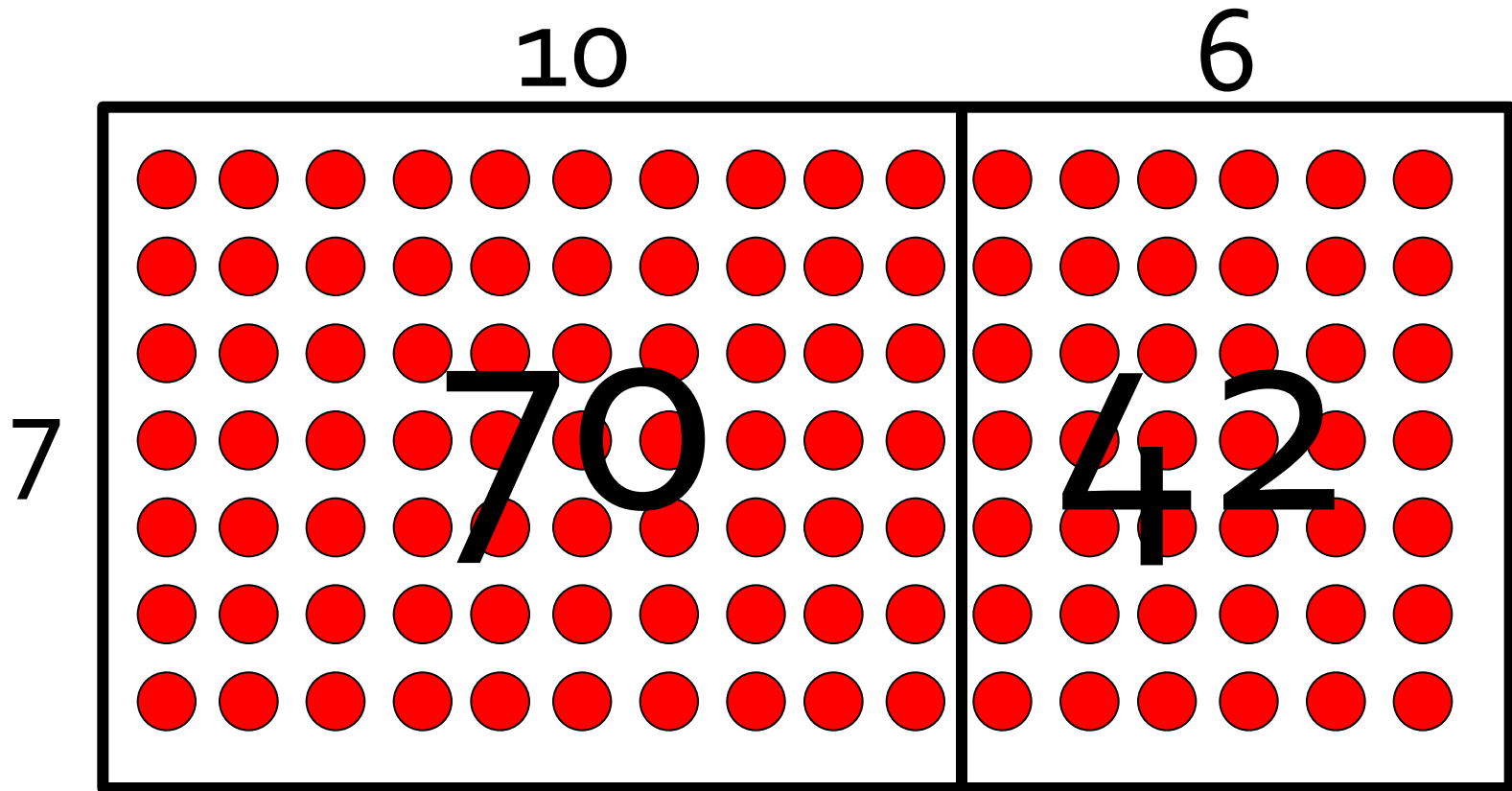
x	x	x	x	x	x	x	
x	x	x	x	x	x	x	
x	x	x	x	x	x	x	
x	x	x	x	x	x	x	

$$4 \times 7 = \quad 7 + 7 + 7 + 7 = \quad 28$$



Tell me what you see.
How would you describe them using number sentences?

Arrays to the Grid Method



$$4 \times 13$$

$$13 \times 4$$

10

3

4

40

12

10

3

4

40

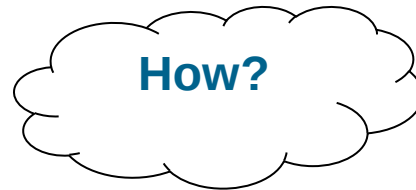
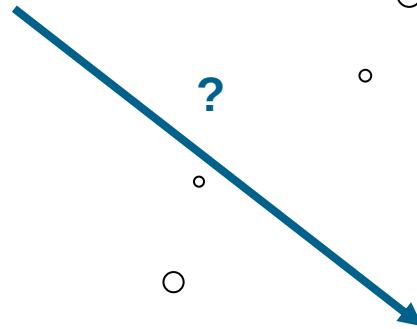
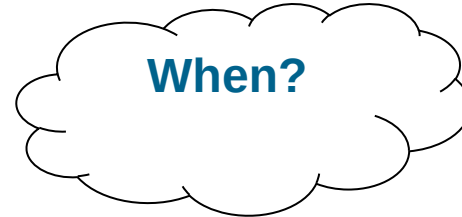
12

$$40 + 12 = 52$$

Progressing towards the standard algorithm

	10	8
10	100	80
3	30	24

	10	8
10	100	80
3	30	24



		1	8			
	×	1	3			
	1	8	0			
		5	4			
	2	3	4			

$$23 \times 8 = 184$$

$\times$	20	3
8	160	24

$$\begin{array}{r} 23 \\ \times 8 \\ \hline 24 \\ 160 \\ \hline 184 \end{array}$$

$$22 \times 18 =$$

$\times$	20	2
10	200	20
8	160	16

$$\begin{array}{r} 22 \\ \times 18 \\ \hline 16 \\ 160 \\ 20 \\ \hline 200 \\ \hline 396 \end{array}$$

Division

Progression in Teaching Division

Mental Skills

Recognise the size and position of numbers

Count back in different steps 2s, 5s, 10s

Halve numbers to 20

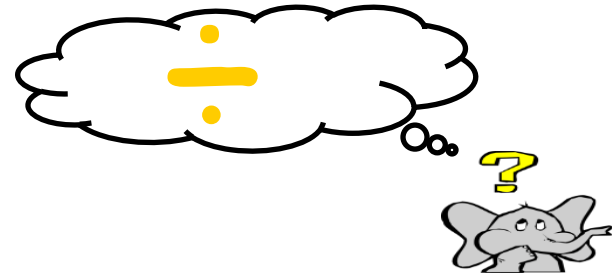
Recognise division as repeated subtraction

Quick recall of division facts

Use known facts to derive associated facts

Divide by 10, 100, 1000 and understanding the effect

Divide by multiples of 10



Models and Images

Counting apparatus

Arrays

100 squares

Number tracks

Numbered number lines

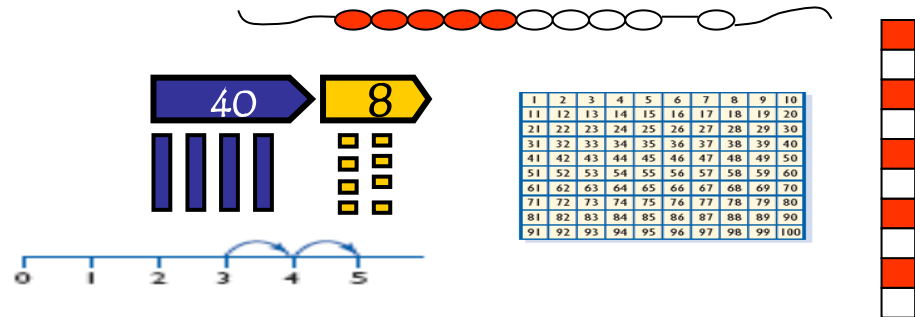
Marked but unnumbered lines

Empty number lines.

Multiplication squares

Models and Images charts

ITPs - Multiplication grid, Number Dials, Grouping, Remainders



Vocabulary

lots of	remainder
factor	divide
Quotient	division
Divisible	divided by
groups of	half
share	
group	
halve	

group groups of
lots of divide
divided by quotient
division factor
remainder divisible
half halve share

Children need to understand the concept of division, that it is:

- **Repeated subtraction**

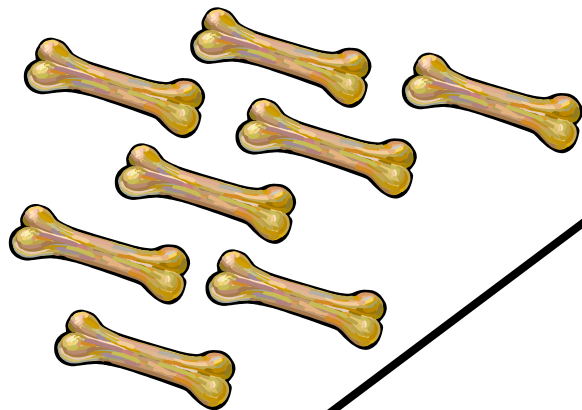
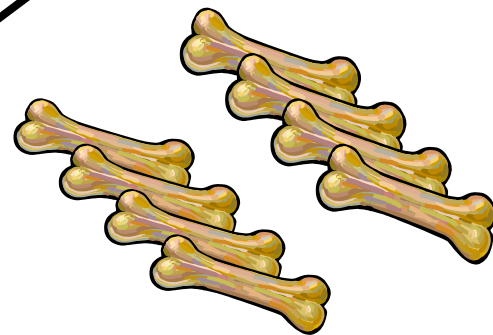
They also need to understand and work with certain principles:

- **Inverse of multiplication**
- **Is not commutative i.e. $15 \div 3 \neq 3 \div 15$**
- **Is not associative i.e. $30 \div (5 \div 2) \neq (30 \div 5) \div 2$**

Sharing

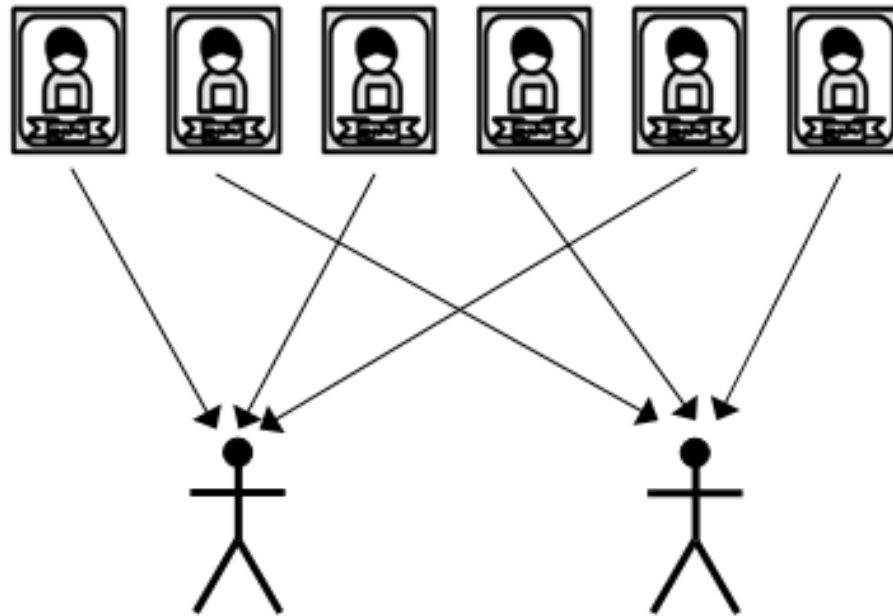


$$8 \div 2$$

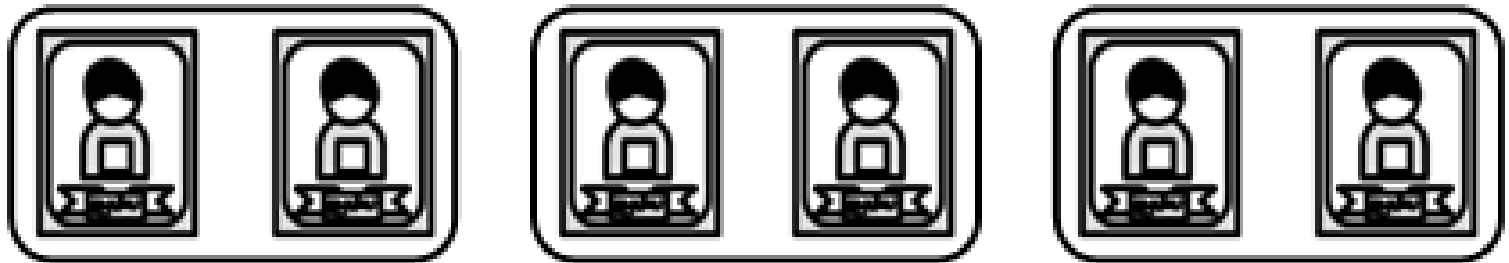


Grouping

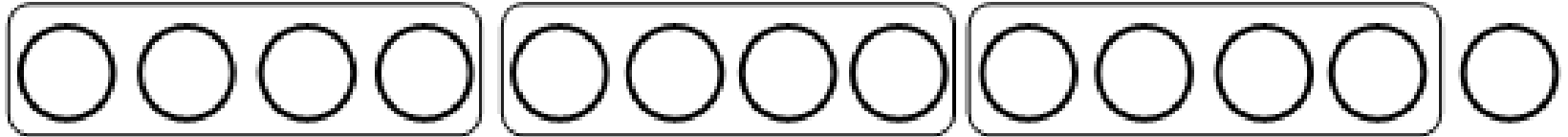
Children naturally start their learning of division as division by sharing, e.g. $6 \div 2$.



To become more efficient, children need to develop the understanding of division as grouping, e.g. $6 \div 2$.

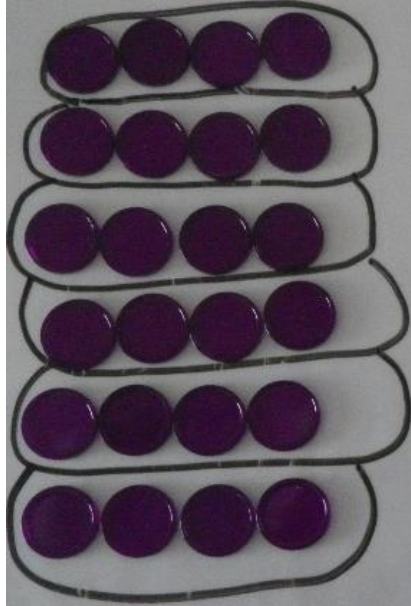


To continue their learning, children need to understand that division calculations sometimes have remainders, e.g. $13 \div 4$.

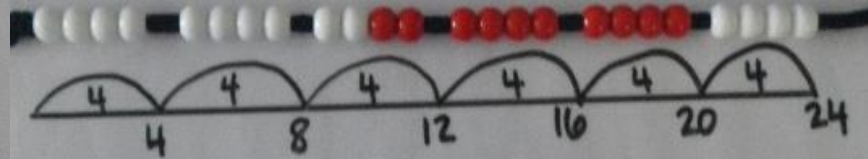
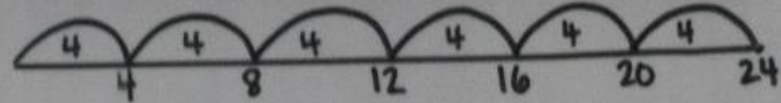


They also need to develop their understanding of whether the remainder needs to be rounded up or down depending on the context.

$$24 \div 4 = \square$$



$$24 \div 4 = \boxed{6}$$



1. $42 \div 3 =$

2. 10×3

I know I have enough to make 10 groups of 3.

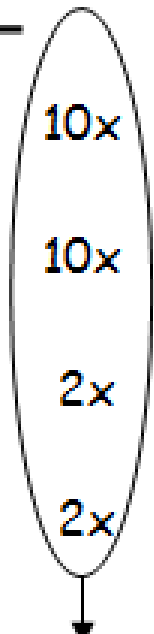
3. 10×3

I have 10 groups of 3 added to 4 groups of 3.
This makes 14 groups
 $42 \div 3 = 14$

Recall of multiplication tables helps make this method more efficient, e.g. $72 \div 3$.

$$\begin{array}{r} 24 \\ 3 \overline{) 72} \\ - 30 \\ \hline 42 \\ - 30 \\ \hline 12 \\ - 6 \\ \hline 6 \\ - 6 \\ \hline 0 \end{array}$$

Answer :



1x	3
2x	6
5x	15
10x	30

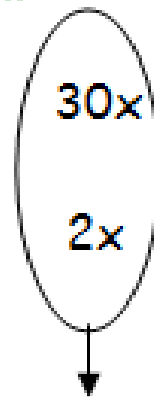
Children should write key facts in a menu box. This will help them in identifying the largest group they can subtract in one chunk.

e.g. $196 \div 6$

$196 \div 6$

$$\begin{array}{r} \text{32 r 4} \\ 6 \overline{) 196} \\ \underline{- 180} \\ 16 \\ \underline{- 12} \\ 4 \end{array}$$

Answer :

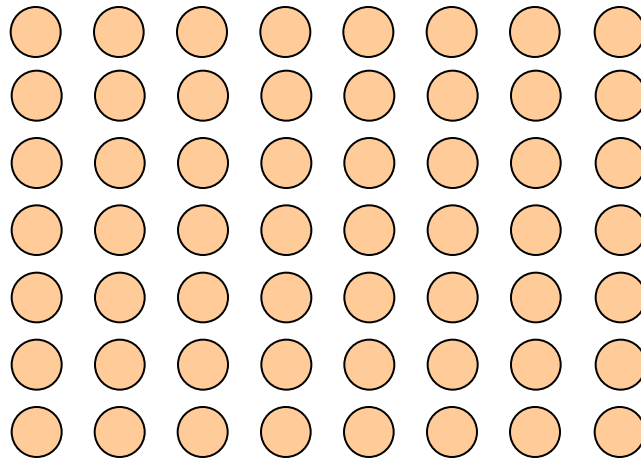
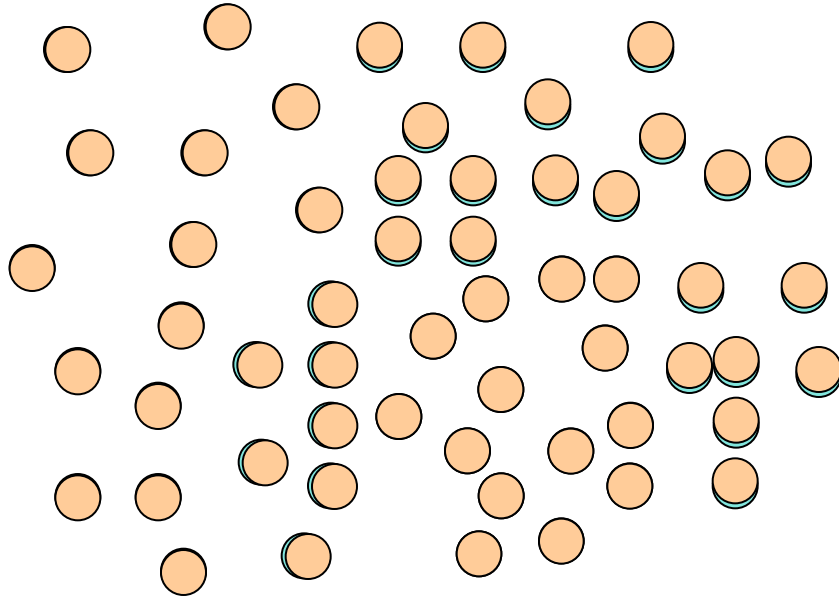


1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

The key facts in the menu box should be extended to include 4x and 20x.

32 remainder 4 or 32 r 4

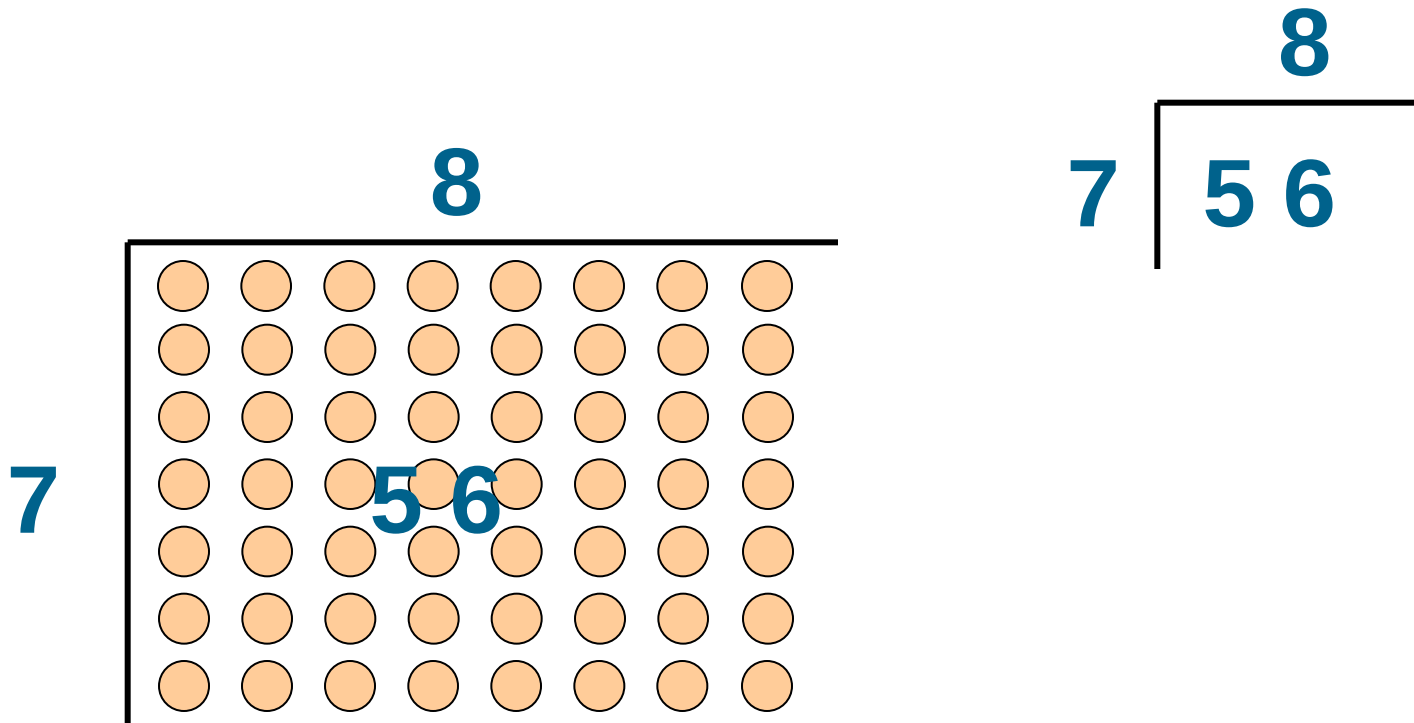
An image for $56 \div 7$



- I move the counters by thinking about how many groups of 7 can I make (grouping)
- But the image helps me to see that 56 shared into 8 groups would mean 7 in each group (sharing)

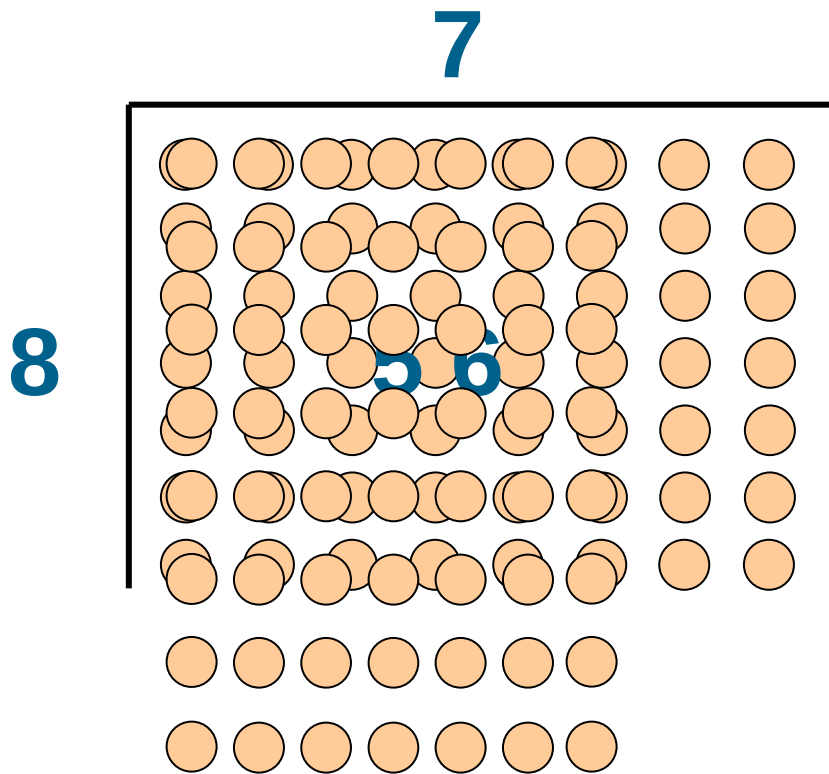
An image for $56 \div 7$

The array is an
image for division



An image for $56 \div 7$ or $56 \div 8$

The array is an
image for division

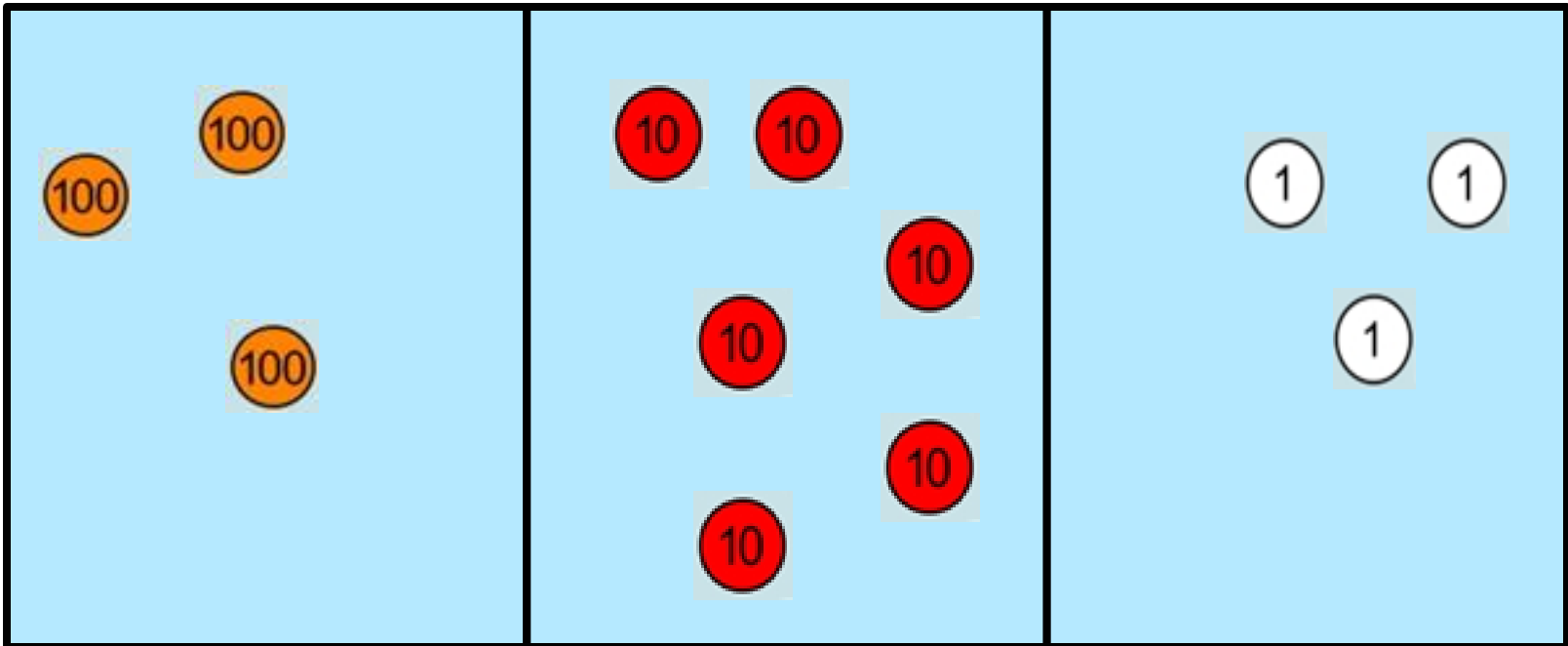


$$\begin{array}{r} 8 \\ 7 \overline{) 56} \end{array}$$

$$\begin{array}{r} 7 \\ 8 \overline{) 56} \end{array}$$

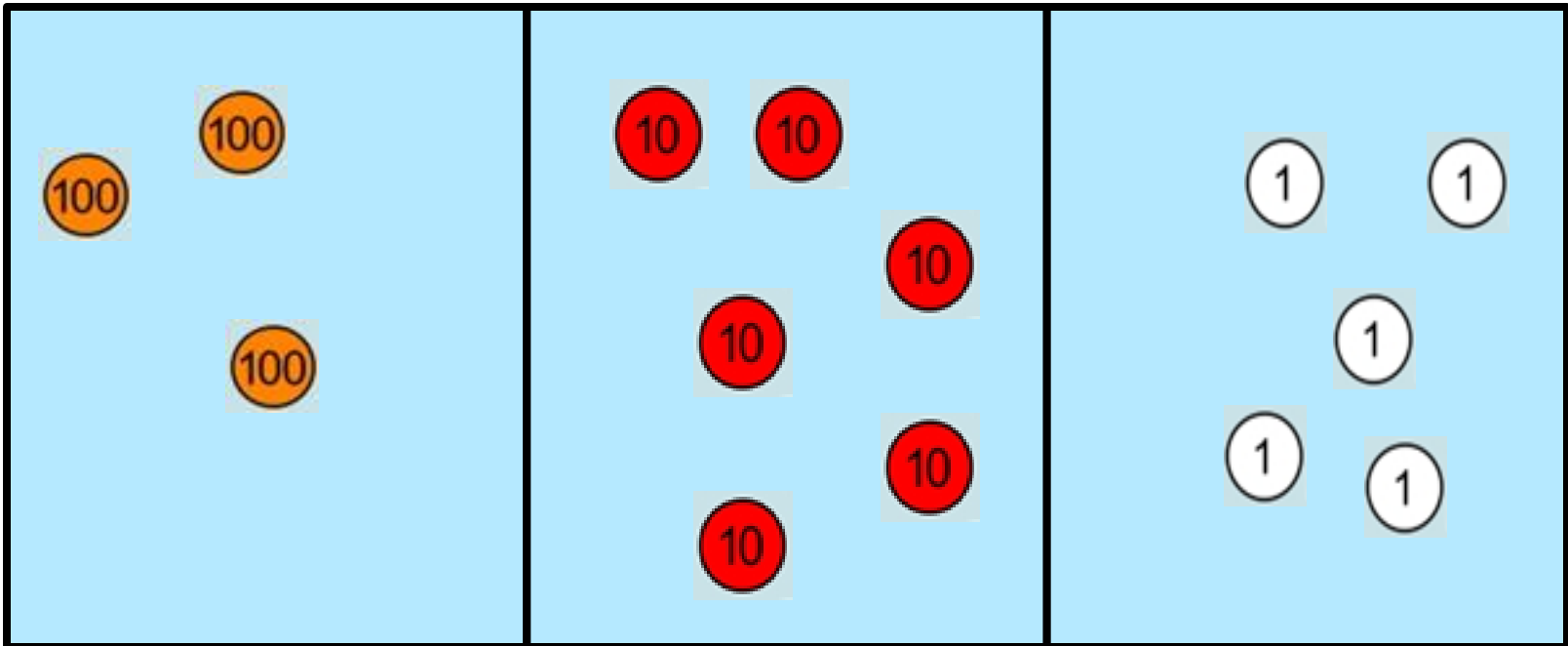
$$363 \div 3 =$$

$$\begin{array}{r} 121 \\ 3 \overline{) 363} \end{array}$$



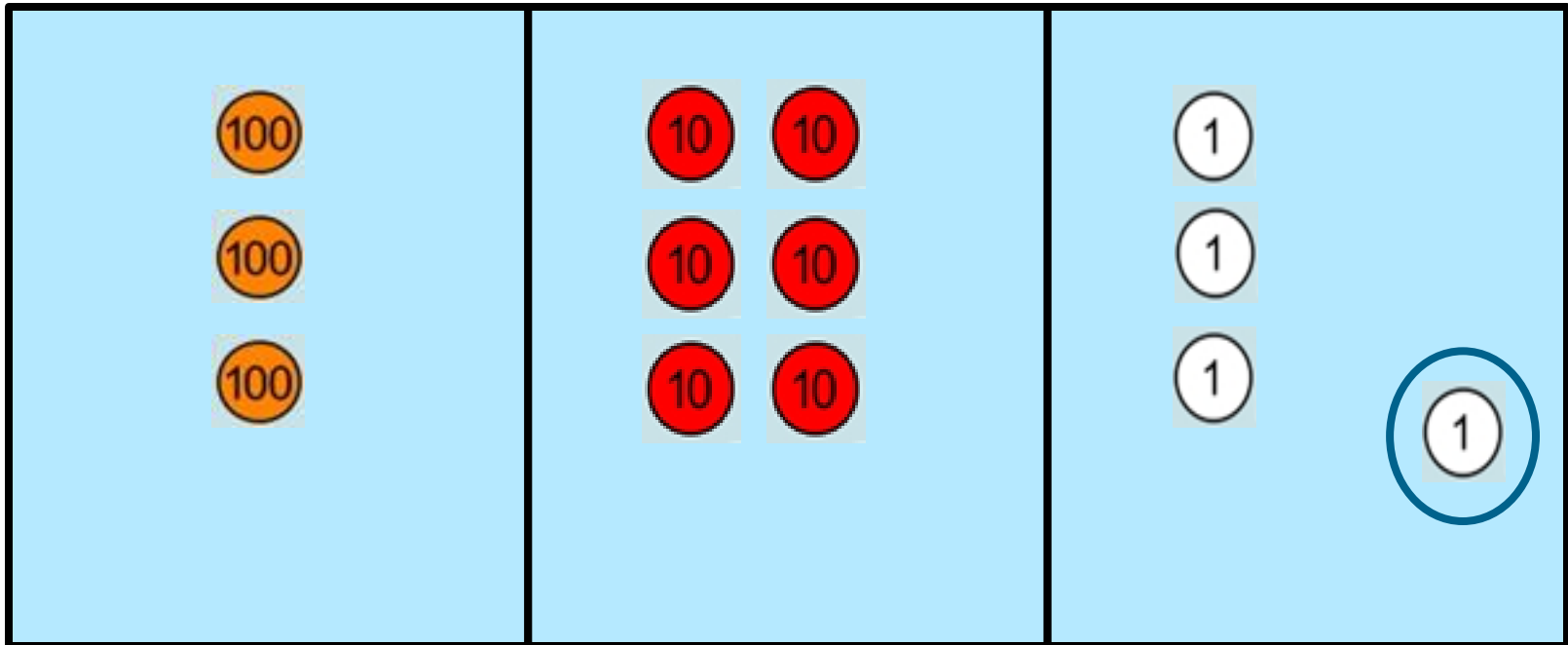
$$364 \div 3 =$$

$$3 \overline{) 364}$$



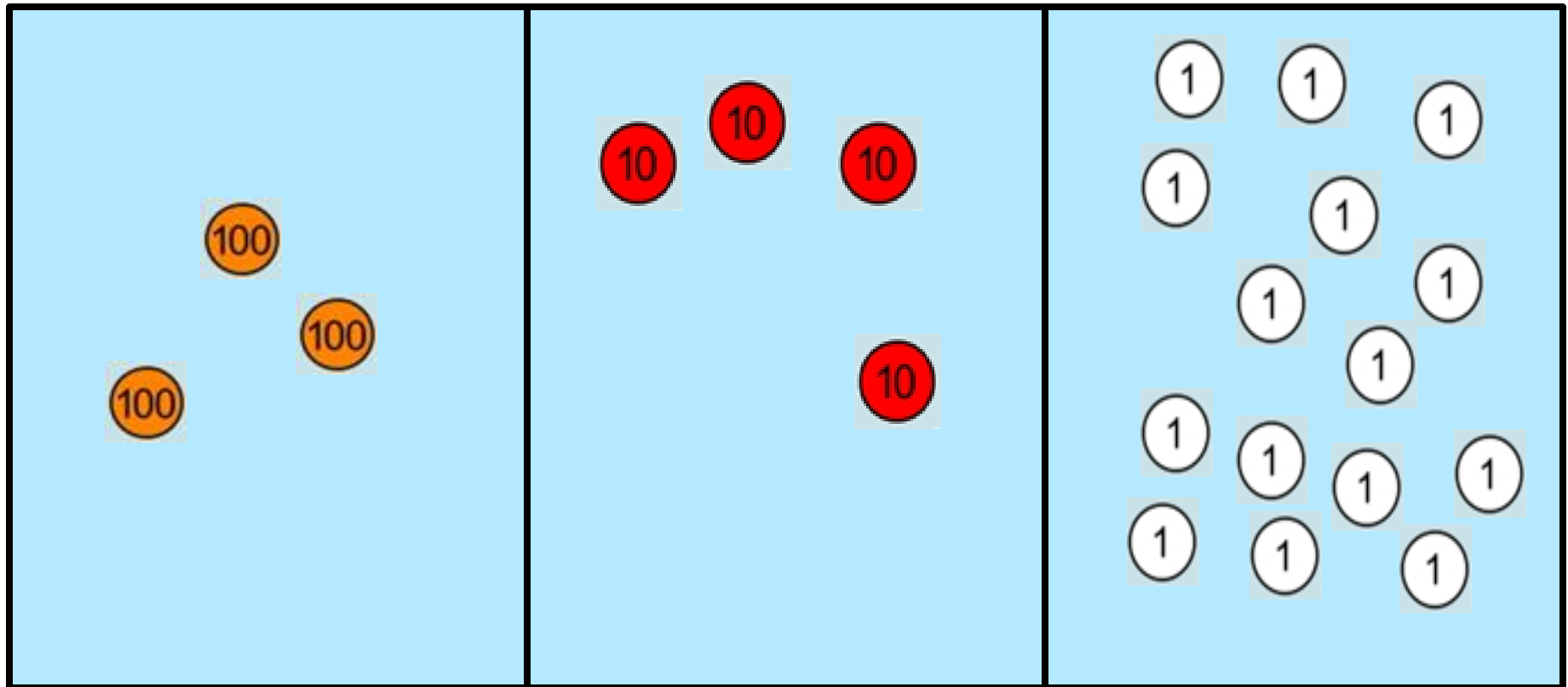
$$364 \div 3 =$$

$$\begin{array}{r} 121 \text{ rem } 1 \\ 3 \overline{) 364} \end{array}$$



$$345 \div 3 =$$

$$\begin{array}{r} 115 \\ 3 \overline{) 345} \\ \underline{3} \\ 4 \\ \underline{3} \\ 5 \end{array}$$



$$6 \overline{) \begin{array}{r} 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{array}} \begin{array}{r} 2 \\ 3 \end{array}$$

